

1. Kittel p. 102 problem 1

1. Monatomic linear lattice. Consider a longitudinal wave

$$u_s = u \cos(\omega t - sKa)$$

which propagates in a monatomic linear lattice of atoms of mass M , spacing a , and nearest-neighbor interaction C .

(a) Show that the total energy of the wave is

$$E = \frac{1}{2} M \sum_s (du_s/dt)^2 + \frac{1}{2} C \sum_s (u_s - u_{s+1})^2 .$$

where s runs over all atoms.

(b) By substitution of u_s in this expression, show that the time-average total energy per atom is

$$\frac{1}{4} M \omega^2 u^2 + \frac{1}{2} C (1 - \cos Ka) u^2 = \frac{1}{2} M \omega^2 u^2 ,$$

where in the last step we have used the dispersion relation (9) for this problem.

This is not just time-average, because energy is constant during the motion, and equals, per atom the maximum kinetic energy of the atom (monatomic only).

2. Kittel p. 103 problem 3

3. Basis of two unlike atoms. For the problem treated by (18) to (26), find the amplitude ratios u/v for the two branches at $K_{\max} = \pi/a$. Show that at this value of K the two lattices act as if decoupled: one lattice remains at rest while the other lattice moves.

3. Kittel p. 103 problem 5

5. Diatomic chain. Consider the normal modes of a linear chain in which the force constants between nearest-neighbor atoms are alternately C and $10C$. Let the masses be equal, and let the nearest-neighbor separation be $a/2$. Find $\omega(K)$ at $K = 0$ and $K = \pi/a$. Sketch in the dispersion relation by eye. This problem simulates a crystal of diatomic molecules such as H_2 .

If you like, you can solve for the complete dispersion relation, but it is simpler just to solve the special cases indicated.

4. Kittel p. 103 problem 6

- 6. Atomic vibrations in a metal.** Consider point ions of mass M and charge e immersed in a uniform sea of conduction electrons. The ions are imagined to be in stable equilibrium when at regular lattice points. If one ion is displaced a small distance r from its equilibrium position, the restoring force is largely due to the electric charge within the sphere of radius r centered at the equilibrium position. Take the number density of ions (or of conduction electrons) as $3/4\pi R^3$, which defines R .
- (a) Show that the frequency of a single ion set into oscillation is $\omega = (e^2/MR^3)^{1/2}$.
- (b) Estimate the value of this frequency for sodium, roughly. (c) From (a), (b), and some common sense, estimate the order of magnitude of the velocity of sound in the metal.