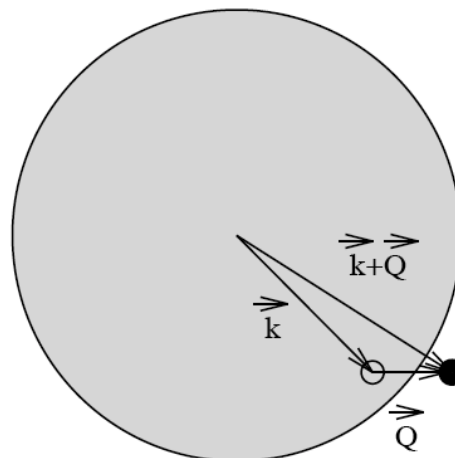


1. Kittel, Ch6 problem 1, plus extra. **(a)** Show that the kinetic energy of a three-dimensional gas of N free electrons at 0K is $3N\epsilon_F/5$. **(b)** Compute the value for an ordered Cu-Al 50/50 alloy with valence assignments 1 for Cu and 3 for Al. The alloy crystal structure is CsCl (simple cubic lattice, two atoms per cell) with lattice constant $a=2.91 \text{ \AA}$. (

2. Kittel, Ch6, problem 5. Liquid ^3He . The atom ^3He has spin $1/2$ and is a Fermion. The density of liquid He is 0.081 g/cm^3 near absolute zero. Calculate the Fermi wave-vector, the Fermi energy and the Fermi temperature.

3. The simplest excited state of the non-interacting electron system is an “**electron-hole pair**” **excitation**. The ground state is a filled “Fermi sea” shown shaded. This “sea” contains N electrons ($N/2$ with spin up, $N/2$ with spin down.) The excited states might involve moving an electron out of the sea and flipping its spin in the process. That would be a “spin-flip” excitation. But the most common excitation has no spin flip. For a fixed \mathbf{Q} (shown in the picture) there are a finite number of such excitations. Here we consider only states that conserve spin. The electron is moved from \mathbf{k} to $\mathbf{k}+\mathbf{Q}$ without changing its spin. **(a)** Show that, for $|\vec{Q}| > 2k_F$, the number of such excitations is N . **(b)** For $|\vec{Q}| < 2k_F$, the number of such excitations is



$3(|\vec{Q}|/2k_F)\left(1 - |\vec{Q}|^2/12k_F^2\right)N$. Use a simple argument for small $|\vec{Q}|$ to argue that the approximate answer is $3(|\vec{Q}|/2k_F)N$.

4. Charge density. An appropriately anti-symmetrized eigenstate of the non-interacting electron gas can be written in various ways, for example:

$$\Psi(r_1, \dots, r_N) = \sqrt{\frac{1}{N!}} \sum_P (-1)^P \prod_{i=1}^N \psi_{P(i)}(r_i) = |1_1, 1_2, \dots, 1_N, 0_{N+1}, \dots\rangle$$

where the occupied single-particle ortho-normal states ψ_i are labeled from $i = 1$ to N .

This state is normalized by $\int dr_1 \dots \int dr_N |\Psi(r_1, \dots, r_N)|^2 = 1$. If the single-particle energies ϵ_i of these states are the lowest possible, then this is the ground state. $P(i)$ means a permutation of the indices i , and P in the exponent is the “order” (+ or - 1 for even or odd) of the permutation. This state is referred to as a “single” “Slater determinant” state. The notation r_i is short for coordinate \vec{r}_i and spin σ_i . The integral dr_i is a 3-d spatial integral $d\vec{r}_i$, and a spin trace (1 if spins are parallel, 0 otherwise.) The electron charge density is defined as $\rho(r_1) = C \int dr_2 \dots \int dr_N |\Psi(r_1, \dots, r_N)|^2$. Show that the charge density of a

single Slater determinant eigen-state is $\rho(r) = C' \sum_{i=1}^N |\psi_i(r)|^2$. (This is not necessarily the ground state.) Why should C' be chosen to be 1? What is therefore the right choice for C ?