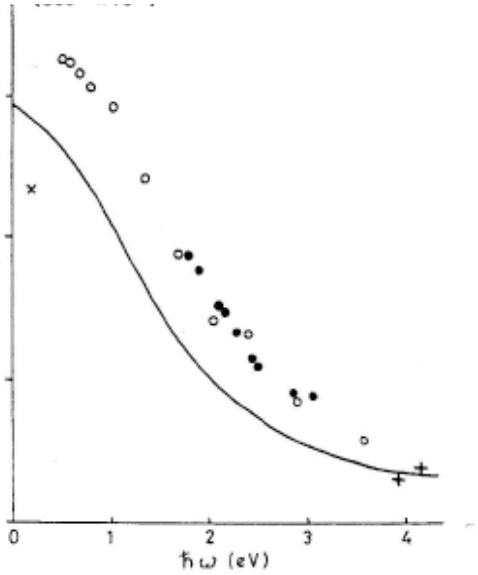
PHY 472 Problem set #8 Due Friday Nov. 18, 2011

1. The resistivity of pure metallic sodium at 300K is 4.6 μΩcm. What is the corresponding relaxation time and mean free path?

2. The figure shows the measured ac conductivity ($\sigma(\omega)$) of liquid mercury at room temperature. The units on the vertical scale are not important. You can ignore the solid line and just consider the data points. The density is 13.6 g/cm^3. You can assume two free electrons per atom. The measured resistivity is 85 μΩcm. From this information, make a free electron model and compute the corresponding mean free path and relaxation rate $1/\tau$. Is this consistent with the figure? The data may roughly be described by the Drude model.



3. Here is the usual model for a solid with dilute "substitutional" impurities.
"Substitutional" means that the impurities replace a particular atom type and sit where the regular atom would ordinarily be. For example, NbN is an "intermetallic" compound, a metal, and an important superconductor. It might be intentionally alloyed with 2% of Zr. It would be a good first approximation that the Zr impurities sit only on the Nb sites, and otherwise have random locations. These impurities will scatter the Bloch electrons of the NbN host. Suppose the Nb atom at the origin is replaced by a Zr. The perturbation felt by the electrons is a local potential energy $V_1(\mathbf{r})$. The total perturbation felt by the electrons is then

$$H' = V_{\text{tot}}(\vec{r}) = \sum_i^{\text{imp}} V_1(\vec{r} - \vec{R}_i)$$

with the impurity sites $\mathbf{R}_i$ randomly distributed over the Nb sublattice.
(a) Show that the matrix element of the perturbation, $\langle \mathbf{k}'|V_{\text{tot}}|\mathbf{k}\rangle$, between host Bloch states $|\mathbf{k}\rangle$, can be written as the matrix element $\langle \mathbf{k}'|V_1|\mathbf{k}\rangle$ for an impurity at the origin, multiplied by an impurity structure factor

$$S(\vec{k} - \vec{k}') = \sum_i^{\text{imp}} e^{i(\vec{k}-\vec{k}')\cdot\vec{R}_i}$$

The sum over phases $\exp(i\phi)$ is a 2d random walk, in the complex plane, of steps of unit length.
(b) The scattering probability per unit time for a Bloch electron $|\mathbf{k}\rangle$ to go to a different Bloch state is given by the Fermi golden rule:

$$1/\tau_k = \frac{2\pi}{\hbar} \sum_{k'} \left| \left\langle \vec{k} \middle| V_{\text{tot}} \middle| \vec{k}' \right\rangle \right|^2 \delta(\varepsilon_k - \varepsilon_{k'})$$

This involves the squared impurity structure factor. Show that this squared structure factor equals N_{imp}, the number of impurities, independent of **k,k'**.

(c) It is also true that the matrix element for a bulk Bloch state ($\psi_{\mathbf{k}}$ normalized to 1 over the whole volume) to couple to another Bloch state $\psi_{\mathbf{k}'}$ via a single impurity atom, $\langle \mathbf{k}'|V_1|\mathbf{k}\rangle$, is has weight 1/N where N is the number of atoms in the whole crystal. And it is true that (as mentioned in class) in a metal, only states near the Fermi surface need to be considered in the conduction process. Thus a formula for the quasiparticle relaxation rate by impurity scattering is

$$1/\tau_k = \frac{2\pi}{\hbar}\frac{N_{\text{imp}}}{N}\frac{1}{N}\sum_{k'}\left|V_{kk'}\right|^2\delta(\varepsilon_k - \varepsilon_{k'}); \qquad 1/\tau = \frac{2\pi}{\hbar}\frac{N_{\text{imp}}}{N}\overline{\left|V_{kk'}\right|^2}D(\varepsilon_F)$$

where $V_{kk'}$ is $\langle k|V1|k'\rangle$ with the factor 1/N removed (ψ_k is now normalized to a unit cell). The notation $1/\tau$ means $1/\tau_k$ averaged over the states on the Fermi surface (the surface in k-space of states at the Fermi energy.) In a free electron model for metallic Na, suppose that there are potassium (K) impurities at the level of 2%, and that the matrix element $\langle \mathbf{k}'|V_1|\mathbf{k}\rangle$ for scattering from one of these is independent of **k,k'** and has the value 1 eV times Compute the corresponding mean free path of an electron at the Fermi level.