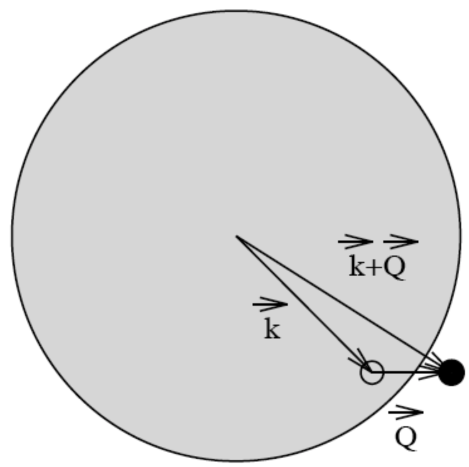
Physics 555 Fall 2011 Problem set #5 Due Friday Oct. 14

1. Kittel, Ch6 problem 1, plus extra. **(a)** Show that the kinetic energy of a three-dimensional gas of N free electrons at 0K is $3N\varepsilon_F/5$. **(b)** Compute the value for an ordered Cu-Al 50/50 alloy with valence assignments 1 for Cu and 3 for Al. The alloy crystal structure is CsCl (simple cubic lattice, two atoms per cell) with lattice constant a=2.91 Å. **(c)** Each electron has spin ½ and magnetic moment μ_B = 1 Bohr magneton. Find the formula for the kinetic energy of an electron gas with (1+ε) N/2 up spins and (1-ε) N/2 down spins, in terms of the Fermi energy of the unpolarized gas, where |ε|< ½. **(d)** Taylor-expand for small ε. Remember that the energy can be written as $U=U_0 + n\chi^{-1}M^2/2$. What do you get for the susceptibility χ?

2. Kittel, Ch6, problem 5. Liquid ^{3}He. The atom ^{3}He has spin ½ and is a Fermion. The density of liquid He is 0.081 g/cm3 near absolute zero. **(a)** Calculate the Fermi wave-vector, the Fermi energy and the Fermi temperature. **(b)** Calculate the pressure that would be needed to maintain this density (at T=0) if ^{3}He were a non-interacting Fermi gas. **(c)** What holds ^{3}He in the liquid state at p=T=0?

3. The simplest excited state of the non-interacting electron system is an **"electron-hole pair" excitation.** The ground state is a filled "Fermi sea" shown shaded. This "sea" contains *N* electrons (*N/2* with spin up, *N/2* with spin down.) The excited states might involve moving an electron out of the sea and flipping its spin in the process. That would be a "spin-flip" excitation. But the most common excitation has no spin flip. For a fixed ***Q*** (shown in the picture) there are a finite number of such excitations. Here we consider only the spin-conserving excitations where the electron is moved from **k** to **k+Q** without altering its spin. **(a)** Show that, for $|\vec{Q}| > 2k_F$, the number of such excitations is *N*. **(b)** For $|\vec{Q}| < 2k_F$, the number of such excitations is $3(|\vec{Q}|/2k_F)(1-|\vec{Q}|^2/12k_F^2)N$. Use a simple argument for small $|\vec{Q}|$ to argue that the approximate answer is $3(|\vec{Q}|/2k_F)N$.



4. Charge density. An appropriately anti-symmetrized eigenstate of the non-interacting electron gas can be written in various ways, for example:

$$\Psi(r_1,\cdots,r_N) = \sqrt{\frac{1}{N!}}\sum_P (-1)^P \Pi_{i=1}^N \psi_{P(i)}(r_i) = |1_1,1_2,\cdots,1_N,0_{N+1},\cdots\rangle$$

where the occupied single-particle ortho-normal states ψ_i are labeled from i = 1 to N. This state is normalized by $\int dr_1 \cdots \int dr_N |\Psi(r_1,\cdots,r_N)|^2 = 1$. If the single-particle energies ε_i of these states are the lowest possible, then this is the ground state. P(i) means a permutation of the indices i, and P in the exponent is the "order" (+ or – 1 for even or odd) of the permutation. This state is referred to as a "single" "Slater determinant" state. The notation r_i is short for coordinate $\vec{r}_i$ and spin σ_i. The integral dr_i is a 3-d spatial integral $d\vec{r}_i$, and a spin trace (1 if spins are parallel, 0 otherwise.) The electron charge density is defined as $\rho(r_1) = C\int dr_2 \cdots \int dr_N |\Psi(r_1,\cdots,r_N)|^2$. Show that the charge density of a single Slater determinant eigen-state is $\rho(r) = C'\sum_{i=1}^{N} |\psi_i(r)|^2$. (This is not necessarily the ground state.) Why should C' be chosen to be 1? What is therefore the right choice for C?