1 Practice Problems for Midterm Exam 1

Problem 1

The Figure 1 depicts velocity as a function of time for a short run. Find:

a) The acceleration at \( t = 5 \) seconds.
b) The acceleration at \( t = 15 \) seconds.
c) The acceleration at \( t = 25 \) seconds.
d) The total distance covered in the 50 second run.
e) The average velocity for the 50 second run.

![v-t graph](image1.png)

Figure 1: \( v-t \) graph

Solution

a) Acceleration is just the slope of the graph at that point. Thus at \( t = 5 \),
\[
a = \frac{8 \text{ m/s} - 0}{10 \text{ s} - 0} = \frac{4 \text{ m}}{5 \text{ s}^2} = 0.8 \text{ m/s}^2.
\]
b) \( a = 0 \).
c) \[a = \frac{-4 - 8 \text{ m/s} \cdot 30 - 20 \text{ s}^2}{30 - 20 \text{ s}^2} = -\frac{6 \text{ m}}{5 \text{ s}^2} = -1.2 \text{ m/s}^2\]
d) Let us calculate the position in parts. Remember position is just the area under the velocity curve.
\[
x(t = 10) = \frac{1}{2} \cdot 8 \text{ m/s} \cdot 10 \text{ s} = 40 \text{ m}
\]
\[
x(t = 20) = x(t = 10) + 10 \text{ s} \cdot 8 \text{ m} = 40 \text{ m} + 80 \text{ m} = 120 \text{ m}
\]
For this part we need to know when the velocity goes to zero. \( 0 - \frac{8 \text{ m}}{5 \text{ s}^2} = -\frac{6 \text{ m}}{5 \text{ s}^2} \cdot (t - 20 \text{ s}) \)
which gives us $t = 20 + \frac{20}{3}$. So $x(t = 30) = x(t = 20) + \frac{1}{2} \cdot \frac{20}{3} \cdot 8 \frac{m}{s} + \frac{1}{2} \cdot \frac{10}{3} \cdot 8 \frac{m}{s} = 140m$. Alternatively $x(t = 30) = x(t = 20) + 8 \frac{m}{s} \cdot 10s + \frac{1}{2} (-1.2 \frac{m}{s^2})(10s)^2 = 140m$.

$$x(t = 50) = x(t = 30) - 10s \cdot 4 \frac{m}{s} - \frac{1}{2} \cdot 10s \cdot 4 \frac{m}{s} = 80m.$$ 

e) $v_{\text{average}} = x(t = 50)/50s = 80m/50s = 1.6 \frac{m}{s}$.

**Problem 2**

On a short hike a student walks 3.0 km 300 South of East then 3.0 km due North then 2.0 km 300 West of North then 4.0 km 370 South of West. Where is she relative to her starting point? Give distance and direction.

**Solution**

$x_{\text{total}} = 3.0 \text{ km} \cos 30^\circ - 2.0 \text{ km} \sin 30^\circ - 4.0 \text{ km} \cos 37^\circ = -1.6 \text{ km}$

$y_{\text{total}} = -3.0 \text{ km} \sin 30^\circ + 3.0 \text{ km} \cos 30^\circ + 2.0 \text{ km} \cos 37^\circ - 4.0 \text{ km} \sin 37^\circ = 0.82 \text{ km}$

$|r| = \sqrt{x^2 + y^2} = 1.8 \text{ km}$ and $\theta = \tan^{-1} \frac{y}{x} = 27^\circ$ north of west

![Diagram]

**Problem 3**

An airplane is heading North with the wind of velocity 30 km/h blowing in the direction of 450 South from West. What is the speed of the plane relative to Earth if its speed relative to the air is 300 km/h ?

**Solution**

$\vec{v}_{\text{airplane,ground}} = \vec{v}_{\text{airplane,air}} + \vec{v}_{\text{air,ground}} = (300 \frac{km}{hr} \hat{j}) + 30 \frac{km}{hr} (- \cos 45^\circ \hat{i} - \sin 45^\circ \hat{j})$ $= -21 \frac{km}{hr} \hat{i} + 279 \frac{km}{hr} \hat{j}$
\[ |v| = 280 \frac{km}{hr} \]

**Problem 4**

Sketch graphs of \( x(t) \), \( v(t) \), and \( a(t) \) for a ball thrown straight up in the air with an initial speed of 5 m/s. Label the graphs and use scales showing appropriate values.

**Solution**
Problem 5

An object which was thrown vertically upward from the edge of the roof of a building is observed to be moving at 10.0 m/s downward as it passes the point from which it was thrown.

a) At what velocity does it hit the ground 80.0 m below the roof?

b) What is the time interval between its passing [downward] the point of the origin and hitting the ground?

c) What is the maximum distance above the point of origin reached by the object?

Solution

a) \( v^2 = v_0^2 + 2ax = 100\frac{m^2}{s^2} + 2(9.8\frac{m}{s^2})(80m) = 1668\frac{m^2}{s^2} \). \( v = 40.8\frac{m}{s} \) downward

b) Using \( v = v_0 - gt \) and then \( t = \frac{-40.8\frac{m}{s} + 10\frac{m}{s}}{-9.8\frac{m}{s^2}} = 3.15s \)

c) Since it was moving down at \( 10\frac{m}{s} \) at the point it was thrown, it had the same initial velocity but upwards. Using \( v^2 = v_0^2 + 2ax \) gives us \( 0 = (10\frac{m}{s})^2 - 2(9.8\frac{m}{s^2})x \) and then \( x = 5.10 \text{ m} \) above the point it was thrown.

Problem 6

A projectile is fired from the top of a cliff at an angle of \( 37^0 \) to the horizontal as shown in Figure 2 with an initial velocity of 150 m/s. It lands on a horizontal plane 2400 m from the base of the cliff.
a) How long is the projectile in the air?

b) How high is the cliff?

c) What is the magnitude and direction of the velocity of the projectile just before it hits the ground?

![Projectile trajectory](image)

**Figure 2: Projectile trajectory**

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**Solution**

a) $x - x_0 = v_{0x}t$

$t = \frac{x - x_0}{v_0 \cos 37^\circ} = 20.0 \frac{m}{s}$

b) $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$0 = h + (150 \frac{m}{s}) \sin 37^\circ)t - \frac{1}{2}gt^2$ gives us

$h = 158m$

c) $v_x = 150 \frac{m}{s} \cos 37^\circ = 120 \frac{m}{s}$

$v_y = 150 \frac{m}{s} \sin 37^\circ - gt = -106 \frac{m}{s}$

$|v| = \sqrt{(v_x^2 + v_y^2)} = 160 \frac{m}{s}$

$\theta = \tan^{-1} \frac{v_y}{v_x} = 41.5^\circ$ south of east

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**Problem 7**

Shown in the figure 3 is a system of blocks connected by a massless string running over a massless pulley. The coefficient of friction on the ramp is $\mu_k = 0.1$. The 8 kg block is sliding down the ramp. Find all the following:

a) The normal force on the sliding block.

b) The acceleration of the system.
c) The tension, $T$, in the cord.

**Solution**

a) Let us call the 8 kg mass $M$ and the 2 kg mass $m$. In a titled coordinate frame so that $y$ is perpendicular to the surface gives us $\sum F_y = 0 = N - Mg \cos \theta$ and then $N = Mg \cos \theta = 8 kg \times (9.8 m/s^2) \cos 30^\circ = 67.9$ N

b) The other two equations are $Mg \sin \theta - T - f = Ma$ and $T - mg = ma$. Also remember $f = \mu N = \mu Mg \cos \theta$ Solving these two equations gives us $a = \frac{Mg \sin \theta - mg - \mu Mg \cos \theta}{M + m} = 1.28 m/s^2$

c) $T = mg + ma = 22.2N$

**Problem 8 (Ex.5.25)**

A passenger on a carnival Ferris wheel moves in a vertical circle of radius $R$ with constant speed $v$. Assuming that the seat remains upright during the motion, derive expressions for the force the seat exerts on the passenger at the top of the circle and at the bottom.
Solution

Let us do the top part of the Ferris wheel first. There the normal force acts out of the circle and weight into so \( mg - N_{\text{top}} = \frac{mv^2}{R} \) and then \( N_{\text{top}} = mg - \frac{mv^2}{R} \).

Now the bottom of the Ferris wheel. Here normal force acts into the circle and weight acts out of the circle and then \( N_{\text{bottom}} - mg = \frac{mv^2}{R} \) and then \( N_{\text{bottom}} = mg + \frac{mv^2}{R} \).