Physics 125: Classical Physics A

1 Practice Problems for Midterm Exam 2 with solutions

Problem 1

A girl throws a water filled balloon of mass 0.1 kg at an angle 45^o above the horizon. The highest point in the trajectory is reached after 0.86 seconds.

(a) What is the vertical component of the initial velocity? (5pts)

(b) What is the initial speed of the balloon? (5pts)

(c) The horizontal component of the balloon's velocity is directed towards a car that is approaching the girl at a constant speed of 8 m/s. If the balloon is to hit the car at the same height at which it leaves her hand, what is the maximum distance the car can be from the girl when the balloon is thrown? Ignore air resistance, and I highly recommend not to try this at home. (10pts)

(d) How much work did the girl do launching the balloon? (5pts)

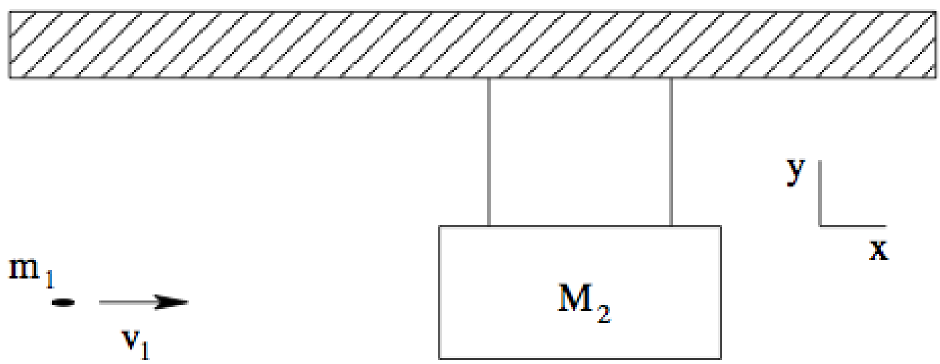
Solution

a)
At maximum height $v_y = 0 = v_{0y} - gt$
Then $v_{0y} = gt = 8.42\frac{m}{s}$
b)
$v_{0y} = v_0 \cos\theta$
$v_0 = \frac{v_{0y}}{\cos\theta} = 11.9\frac{m}{s}$
c)
First let us consider the range of the water balloon.
Thus $R = v_0 t \cos\theta$ and $0 = v_0 t \sin\theta - \frac{1}{2}gt^2$.
Solving for R gives us $R = \frac{2v_0^2 \sin\theta\cos\theta}{g}$ and $t = \frac{2v_0 \sin\theta}{g}$
The car's position is given by $x_{car} = d - v_{car}t = d - \frac{2v_{car}v_0\sin\theta}{g}$
Letting $x_{car} = R$ gives us $d = \frac{2v_0\sin\theta(v_{car}+v_0\cos\theta)}{g} = 28.2m$
d)

$W = \Delta K = \frac{1}{2}mv_0^2 - 0 = 7.08J$

Problem 2

A bullet of mass $m_1 = 10$ g is fired with velocity $\vec{v_1} = 300\,\hat{\imath}\,\frac{m}{s}$ toward a stationary block of wood of mass $M_2 = 3$ kg, and is very rapidly brought to rest inside the block. The block is initially hanging from the ceiling by virtue of two massless strings as shown.



(a) What is the speed of the bullet and block just after the bullet is stopped (but before the combined mass has time to move an appreciable distance)?

(b) How high do the bullet and block rise relative to their initial height?

Solution

a)
$p_i = m_1\vec{v}_1$
$p_f = (m_1 + M_2)\vec{v}$
$\vec{v} = \frac{m_1}{m_1+M_2}\vec{v}_1 = 0.997\frac{m}{s}\hat{i}$
b)
$E = \frac{1}{2}(m_1 + M_2)v^2 = (m_1 + M_2)gh$
$h = \frac{v^2}{2g} = 5.07cm$

Problem 3

Wayne Gretzky is skating at 13.0 m/s towards a defender in the final national league game. The defender is skating at 5.0 m/s towards Gretzky. Gretzky's weight is 756N and that of the defender is 900N. Immediately after the collision Gretzky keeps moving in his original direction at 1.5 m/s. You can ignore external horizontal forces applied by the ice to the skaters during the collision.

(a) What is the velocity of the defender immediately after the collision?

(b) Calculate the change in the combined kinetic energy of the two players?

Solution

$$m_1 g = 756N, \quad m_2 g = 900N, \quad v_1 = 13.0m/s, \quad v_2 = -5.0m/s$$

(a)

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{where} \quad v_1' = 1.5m/s$$
$$v_2' = 4.7m/s$$

(b)

$$[\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2] - [\frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2] = \Delta K = 6.6kJ \text{ (check numerical value)}$$

Problem 4

The mass m_1=5kg starts at rest on a frictionless surface 5 meters above the ground. It collides elastically with a mass m_2=10 kg (15 kg) once it reaches ground level.

a) What is the speed of m_2 after the collision?
b) What is the maximum height m_1 reaches after the collision?

Solution

a) use energy conservation and momentum conservation!

First get velocity of m_1 before collision:

$$m_1 gh = \frac{1}{2} m_1 v_1^2$$
$$v_1 = \sqrt{2gh} = \sqrt{2 \cdot 5m\ 9.8m/s^2} = 9.9m/s$$

now get velocity of m_2 after elastic collision, use relation between velocities and momentum conservation both with v2i=0!

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \Rightarrow v_{1f} = v_{2f} - v_{1i}$$
$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$
$$\Rightarrow m_1 v_{1i} = m_1 v_{2f} - m_1 v_{1i} + m_2 v_{2f}$$
$$\Rightarrow v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$
$$\Rightarrow v_{2f} = \frac{2 \cdot 5kg}{10kg + 5kg} \cdot 9.9m/s = 6.6m/s$$

(v_{2f}= 4.95 m/s^2)

b) Either calculate v_{1f} and use energy conservation or use energy conservation immediately:

$$v_{1f} = v_{2f} - v_{1i}$$
$$\Rightarrow v_{1f} = 6.6m/s - 9.9m/s = -3.3m/s$$

$$\frac{1}{2} m_1 v_{1f}^2 = m_1 gh'$$
$$\Rightarrow h' = \frac{v_{1f}^2}{2g} = \frac{(3.3m/s)^2}{2 \cdot 9.8m/s^2} = 0.56m$$

Alternatively you can use energy conservation:

$$\frac{1}{2} m_2 v_{2f}^2 + m_1 gh' = m_1 gh$$

Problem 5

A cart of mass $m_1 = 0.200kg$ moves on a horizontal air track with $\vec{v_1} = 0.500\,\hat{\imath}\,\frac{m}{s}$. It collides elastically with a second cart ($m_2 = 0.300\,kg$) initially at rest.

(a) Compute the initial kinetic energy of the system.

(b) Find the final velocities of m_1 and m_2.

(c) If the collision occurs in 0.1 s, what is the average force exerted by car 1 on car 2?

NOTE: As shown in class (and the textbook) for elastic collisions in one dimension we can replace the kinetic energy formula with:

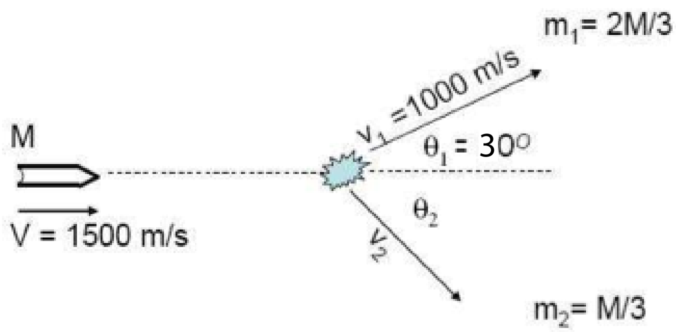
$$v_1 - v_2 = -\left(v_1' - v_2'\right) \tag{1}$$

Solution

a)
$K_i = \frac{1}{2}m_1v_1^2 = 0.0250J$
b)
Using $m_1v_1 = m_1v_1^{'} + m_2v_2^{'}$ and $v_1 - v_2 = -(v_1^{'} - v_2^{'})$, gives us $\vec{v}_1^{\,'} = \frac{m_1-m_2}{m_1+m_2}\vec{v}_1 = -0.100\frac{m}{s}\hat{\imath}$ and $\vec{v}_2^{\,'} = \frac{2m_1}{m_1+m_2}\vec{v}_1 = 0.400\frac{m}{s}\hat{\imath}$
c)
$\vec{F}_{average} = m_2\frac{\Delta\vec{v}_2}{\Delta t} = \frac{m_2\vec{v}_2}{\Delta t} = 1.20N\hat{\imath}$

Problem 6

A rocket of mass $M = 30.0\ kg$ is travelling with velocity $\vec{V} = 1500\,\hat{\imath}\ \frac{m}{s}$ (horizontally) when it explodes into two chunks of mass $m_1 = 20.0\ kg$ and $m_2 = 10.0\ kg$. The mass m_1 is observed to be movingf at an angle $\theta_1 = 30.0^o$ above the horizontal with a speed $|\vec{v_1} = 1000\ \frac{m}{s}|$ just after the explosion.



(a) Find the magnitude and direction of the velocity of m_2, $|v_2|$, and θ_2. [HINT: You may need to know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.]

(b) Do a calculation that allows you to determine whether this explosion was an elastic or inelastic process.

Solution

a)
Before the collision, $p_{i,x} = MV$ and $p_{i,y} = 0$.
After the collision, $p_{f,y} = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$ and $p_{f,x} = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$.
Plugging in $\theta_1 = 30°$ gives us:
$0 = \frac{1}{2} m_1 v_1 - m_2 v_2 \sin\theta_2$ and $MV = \frac{\sqrt{3}}{2} m_1 v_1 + m_2 v_2 \cos\theta_2$
Solving the first equation for $m_2 v_2$ gives us $m_2 v_2 = \frac{m_1 v_1}{\sin\theta_2}$
and this gives us $\tan\theta_2 = \frac{m_1 v_1}{2MV - \sqrt{3} m_1 v_1}$
or $\theta_2 = 0.347$ radians$= 19.9°$.
Then $v_2 = 2943 \frac{m}{s}$

b)
$K_i = \frac{1}{2}MV^2 = 3.38 \times 10^7 J$
$K_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = 5.33 \times 10^7 J$
Since $K_i \neq K_f$, collision is inelastic.

Problem 7

Two blocks of masses $m = 1$kg and $M = 3$kg are held at the distance $d = 10$cm against the force of a compressed spring between them. Then the blocks are released and are pushed apart by the spring. Assume that there is no friction. The spring is massless, has a force constant $k = 200$N/m, and an unstressed length of $l = 15$cm. Find

a) The total kinetic energy of blocks after "collision".

b) Final velocities of blocks.

Solution

a)
$E_i = U_i = \frac{1}{2}k(x - x_0)^2$
$E_f = K_f = \frac{1}{2}k(x - x_0)^2 = 0.25J$
b)
Calling the final velocity of mass m,v, and mass M,V.
By energy conservation: $\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = 0.25J$
By momentum conservation: $mv + MV = 0$
Solving these two equations gives us $v = 0.204\frac{m}{s}\hat{i}$ and $V = -0.612\frac{m}{s}\hat{i}$.

Problem 8

The angle θ through which a bycicle wheel turns is given by $\theta(t) = a + bt^2 - ct^3$ where a, b and c are positive constants such that for t in seconds θ is in radians.

(a) Calculate the angular acceleration of the wheel as a function of time.

(b) At what time is the angular velocity of the wheel instantaneously not changing?

Solution

a)
$\omega = \frac{\partial\theta}{\partial t} = 2bt - 3ct^2$
$\alpha = \frac{\partial^2\theta}{\partial^2 t} = 2b - 6ct$
b)
ω is constant when $\alpha = 0$, this is at $t = \frac{b}{3c}$

Problem 9

Three small blocks, each with mass m, are clamped at the ends and at the center of a massless rod of length L. (The rod is massless, do you think you need to include the moment of inertia of the rod?)

(a) Compute the moment of inertia of the system about an axis perpendicular to the rod and passing through a point 1/4 of the length from one end.

(b) Computer the moment of inertia of the system about at axis perpendicular to the rod and passing through the center of the rod using the "parallel axis theorem" for moment of inertia.

(c) Check that your answer is correct by explicitly calculating the moment of inertia about the axis in (b)

Solution

(a)

$$I_{L/4} = m[L/4]^2 + m[L/4]^2 + m[3L/4]^2 = (11/16)mL^2$$

(b)

$$I_{L/4} = I_{cm} + Md^2 \Longrightarrow I_{cm} = I_{L/4} - (M = 3m)(L/4)^2$$

$$I_{cm} = (11/16)mL^2 - 3m(L/4)^2 = (1/2)mL^2$$

(c)

$$I_{cm} = m(L/2)^2 + m(0)^2 + m(L/2)^2 = (1/2)mL^2$$