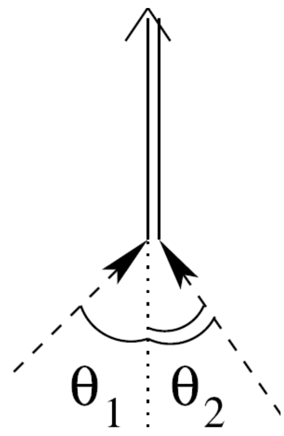
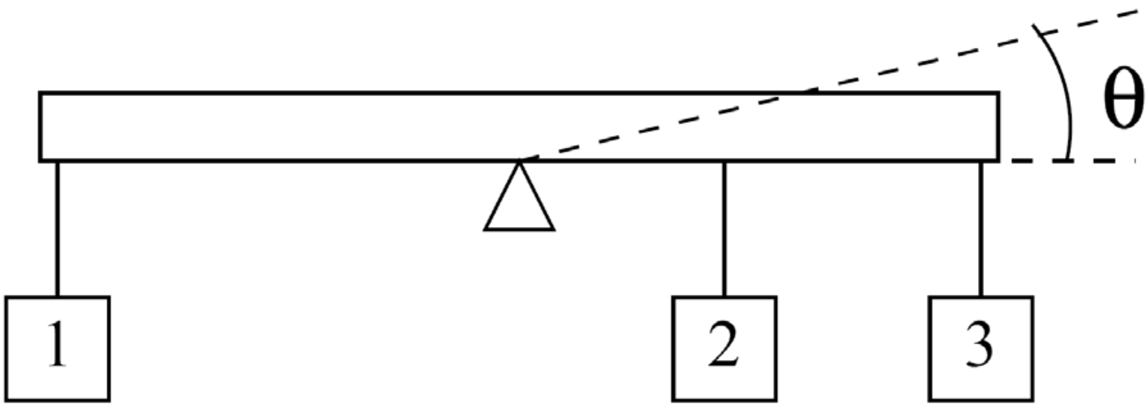
1. Two skiers of mass m_1 and m_2 are skiing on frictionless snow. They descend from the same height h, starting from rest, to a flat surface. The diagram to the right looks down at an event on this flat surface. At this point the skiers have equal speeds ($|\vec{v}_1| = |\vec{v}_2|$) and they collide. They stay together and proceed without friction in the direction shown (due north, the y-direction.) They came from different directions as shown, with θ_1=45° indicating an approach from the south west, and θ_2=37° indicating an approach from the south east.



   a. *(10 pt.)* What is the ratio m_1/m_2?
   b. *(10 pt.)* After colliding, their common speed is 10.0 m/s. What was the speed of the skiers before colliding? [You may assume if you wish that M_1 = 70 kg.]
   c. *(10 pt.)* Prove the equality of their speeds ($|\vec{v}_1| = |\vec{v}_2|$ just **before** the collision) using the ideas of Newtonian mechanics. [Note that the angle φ of the slope is **not** the same everywhere, so the down-hill acceleration ($g \sin \varphi$) **cannot** be assumed constant.]

a. Let x and y directions define the conventional W→E and S→N Cartesian coordinates for the skiers. The total momentum is conserved in the collision. Therefore, in particular, the x component is conserved. The final x-momentum is zero, so the initial x-momentum is zero. This tells us that $m_1v_{1x}= - m_2v_{2x}$. We also have that $v_{1x}=|\mathbf{v}_1|\sin \theta_1$ and $v_{2x}= - |\mathbf{v}_2|\sin \theta_2$; and of course, $|\mathbf{v}_1|=|\mathbf{v}_2|=v_i$. Therefore we get that $m_1/m_2=\sin \theta_2/\sin \theta_1 = (3/5)/(1/\sqrt{2})=0.85$.
b. The final y-momentum is $(m_1 + m_2)(v_f=10\text{ m/s})$. This must equal the initial y-momentum, $m_1v_i\cos\theta_1 + m_2v_i\cos\theta_2$. Dividing both sides by m_2, we get $(m_1/m_2 + 1)v_f=[(m_1/m_2)(1/\sqrt{2})+(4/5)]v_i$. It is not necessary to know the values of the masses m_1 or m_2, just their ratio from part a. The answer is v_i=13.2 m/s.
c. Each skier gains KE equal to the work done by gravity, $mg\Delta y$. Since each skier descended the same $\Delta y=h$, each has $mv^2/2=mgh$. The mass m cancels, and $v=\sqrt{2gh}$ for each skier.



2. A massless bar, of negligible thickness, and length $L = 0.800$ m, is pivoted around a stationary point at its center as shown. Three masses, $M_1 = 2.00$ kg, $M_2 = M_3 = 1.00$ kg, are suspended under gravity as shown, at positions $x_1 = -L/2$, $x_2 = L/4$, and $x_3 = L/2$, relative to an origin at the pivot. The centers of the masses lie below the pivot by a distance Δy which is negligible compared to L (the picture is not to scale). At time $t=0$, the bar is (instantaneously) at rest ($d\theta/dt = 0$) but not necessarily in equilibrium, in a horizontal position ($\theta_0 = 0$.)

a. *(6 pt.)* The y-component of the system's center of mass is $y_{CM} \sim 0$ m. What is the x-component? Choose the origin at the pivot.
b. *(6 pt.)* What is the moment of inertia I about the pivot point of the system of bar and three masses?
c. *(6 pt.)* At time $t = 0$ the system is allowed to begin rotating around the pivot. What is the initial angular acceleration α?
d. *(6 pt.)* How long does it take to rotate until the masses 1 and 3 have displaced by $\Delta y = \pm 0.005$ m? You may assume that the corresponding angle θ is so small that the angular acceleration α remains constant.
e. *(6 pt.)* How much mass must be added or subtracted from M_1 in order for the system to be in equilibrium? (Technically this is an unstable equilibrium).

a. The x-component of the center of mass is $x_{CM}=\sum m_i x_i/\sum m_i$. The denominator is the total mass, 4 kg. Taking the pivot as the origin, the numerator is 2 kg x (-0.4 m) + 1 kg x (+0.2 m) + 1 kg x (0.4 m) = -0.2 kg m. Dividing, we get x_{CM}= -0.0500 m.
b. The moment of inertia is $I = \sum m_i x_i^2 = 2 \text{ kg} \times (-0.4\text{m})^2 + 1 \text{ kg} \times (0.2\text{m})^2 + 1 \text{ kg} \times (0.4\text{m})^2 = 0.52 \text{ kg m}^2$.
c. We need to find the torque around the pivot. This is just the weight 4 kg x 9.8 m/s^2 = 39.2 N, times the moment arm 0.0500 m (the distance from the center of mass to the pivot, or $\tau = 1.96$ Nm. Then $\alpha = \tau/I = 1.96 \text{ Nm}/(0.52 \text{ kg m}^2) = 3.8 \text{ rad/s}^2$.
d. We use $\Delta\theta = (1/2)\,\alpha t^2$ for constant angular acceleration α, and $\Delta\theta = \Delta y/(L/2)$ for the angle in radians. Therefore $\Delta y = L\,\alpha t^2/4 = 0.005$, or t = 0.25s.
e. Since the torque is counterclockwise 1.96 Nm, we need to subtract mass from M_1. A weight of 4.9 N subtracted from the left will give a torque - 1.96 Nm as needed, and this corresponds to subtracting 0.500 kg from M_1.

3. A mass $M = 0.500$ kg oscillates, without friction, with energy E=1.00 J, because of an ideal spring, of constant $k = 200$ N/m. At time $t = 0$, the mass has zero velocity and displacement $x(0) < 0$.

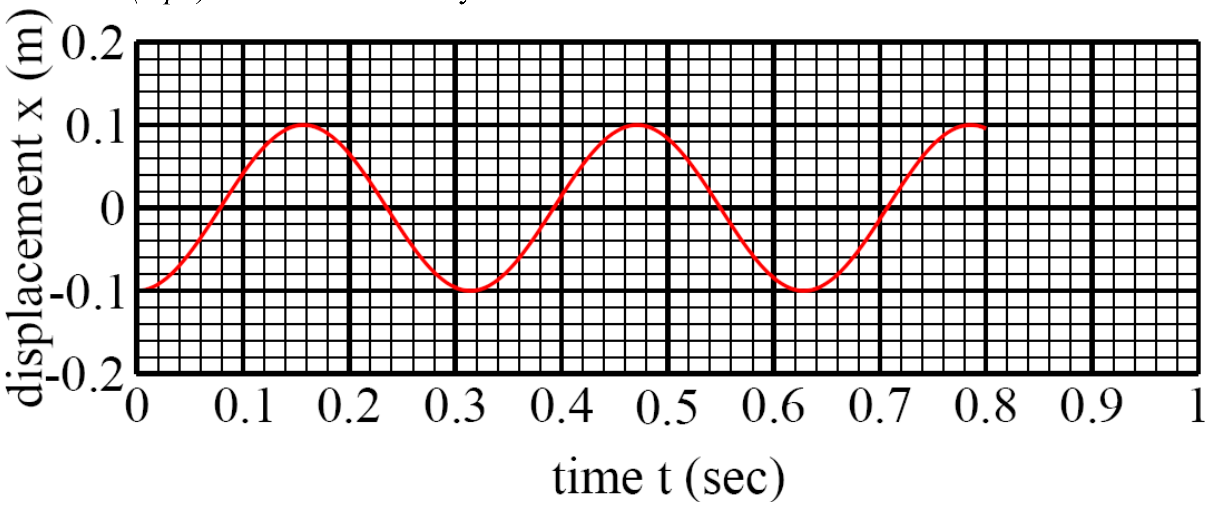
a. *(5 pt.)* What is the maximum velocity of the mass?

b. *(5 pt.)* What is the amplitude of oscillation?

c. *(5 pt.)* What is the phase φ in the conventional formula $x(t)= A\cos(\omega t + \varphi)$?

d. *(10 pt.)* Graph the motion $x(t)$ on the grid below, labeling and numbering the axes, showing times from $t = 0$ to t at least equal to $2T$, where T is the period.

e. *(5 pt.)* What is the velocity at $t = 0.50$ s?



a. Since $E=(1/2)mv_{max}^2 = 1.00$J and m=0.500 kg, v_{max}=2.00 m/s.

b. We will need to know the angular frequency ω which equals $\sqrt{(k/m)} = 20.0$ rad/s. One way to get the amplitude A is from $v_{max} = \omega A$, so A=0.100 m.

c. The easiest way to get φ is first to graph the motion. Since it starts at t=0 with x negative and v=0, it must be just the negative of a cosine function. That means the phase differs from a cosine by π, i.e. $\varphi = \pi$ or $-\pi$ (they are the same.)

d. The graph is shown for exactly two periods. The period $T = 2\pi/\omega = 0.314$ s.

e. The general formula for velocity is the derivative of the general formula given for position, so $v(t=0.5s) = -\omega A\sin(\omega t+\varphi) = -v_{max}\sin(10 \text{ rad} + \pi) = -(2.00 \text{ m/s}) \sin(13.14 \text{ rad}) = -1.09$ m/s. You can see that this is right, at least approximately, from the graph.