PHY131 Spring 2001; Final Exam Answers (Version 2)
14 May, 8:00-10:30am

Remove everything from your work area, except this exam and your calculator. Have your student ID ready. Wait for the word "go" before turning this page and starting the exam. Use the paper provided for all work (use the back sides if necessary). Ask for empty sheets if you need more. Ask permission if you need to leave the room. Do not remove this exam from the room until the exam is over.

Do any 4 of the first 6 problems. Do all 4 of the last 4 problems. All problems are worth 20 points.

Remember to show units and a sensible number of significant figures.

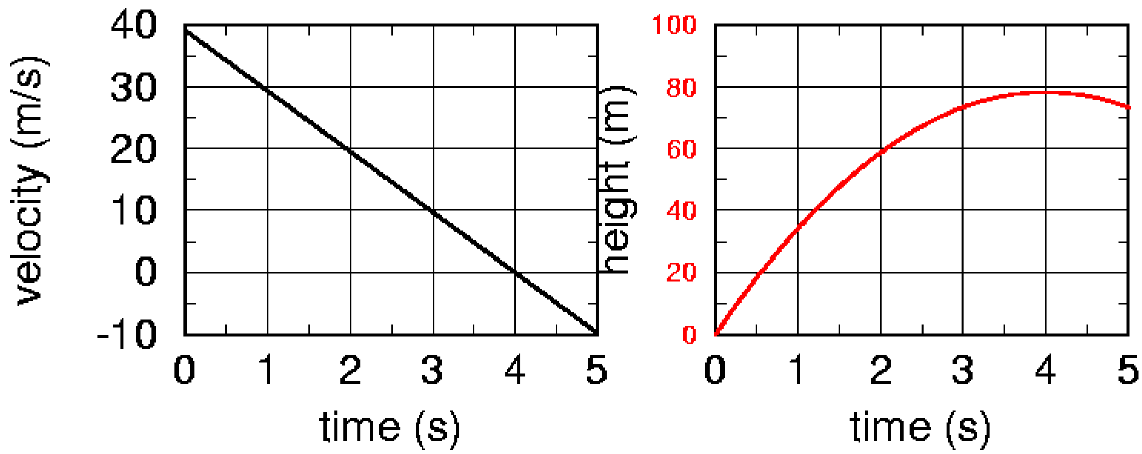
The grid below will be used by graders to record your scores.

	grade	max
problem 1		20
problem 2		20
problem 3		20
problem 4		20
problem 5		20
problem 6		20
problem 7		20
problem 8		20
problem 9		20
problem 10		20
total		160

Possibly useful numbers

$G=6.67 \times 10^{-11}$ Nm^2/kg^2
$\sigma=5.67 \times 10^{-8}$ W/m^2K^4
Average atomic weight of air molecules = 28.2
C(water)=4.186×10^3 J/kg°C
L_v(water)=2.256×10^6 J/kg
L_f(water)=3.34×10^5 J/kg
1 amu = 1.66×10^{-27} kg

1. **(20 points)** An arrow (mass m=0.22 kg) is shot at an angle 45° above the horizontal. Its vertical velocity *versus* time is shown in the left-hand graph (only the first 5 seconds is shown.)

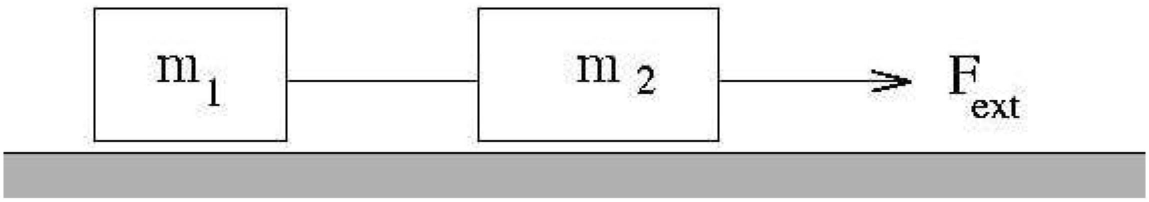


a. **(6 points)** At what time does the arrow hit the ground? Ans: It takes 4.0 s for the vertical velocity to fall from +39.2 m/s to zero. At this point the arrow is at the top of its parabolic trajectory. It takes another 4.0 s to get back to ground, so the answer is 8.0 s.

b. **(6 points)** What is the total horizontal displacement? Ans: The initial vertical velocity is 39.2 m/s, and the angle is 45° above the horizontal, so the horizontal velocity is (39.2 m/s)/tan(45°)=39.2 m/s. In 8.0 s the horizontal displacement is 314. m.

c. **(8 points)** Graph the vertical position (height) *versus* time on the right-hand graph above. Be sure to indicate the scale on the vertical axis.

2. (**20 points –show your work.**) Two masses, m_1=8.00 kg and m_2=14.00 kg, are coupled by a massless cord. They slide on a horizontal surface. An external force F_{ext}=24.0 N is applied to mass m_1 as shown.



a. (**5 points**) Suppose there is no friction. Find the acceleration. Ans: Newton's laws for the two masses are $T=m_1a$ and $F_{ext}-T=m_2a$. Adding these gives $F_{ext}=(m_1+m_2)a$, so $a=F_{ext}/(m_1+m_2)$=24.0 N/22.0 kg=1.09m/s^2.

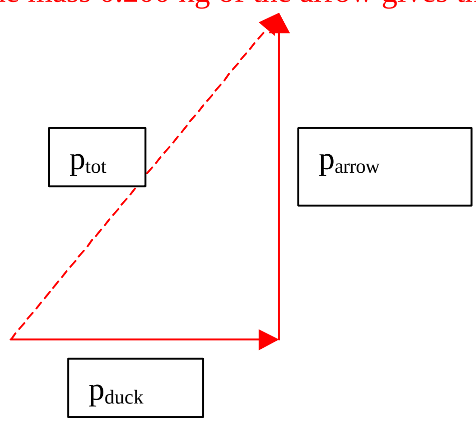
b. (**5 points**) Find the tension in the cord. Ans: $T=m_1a$=8.00 kg x 1.09 m/s^2 =8.73 N.

c. (**10 points**) Now suppose that there is friction, determined by a coefficient *m* of kinetic friction which is the same for each mass. The system slides to the right at constant speed *v*=0.64 m/s. What is the coefficient of friction *m* ? Ans: Newton's laws for the two masses are $T-\mu m_1g=m_1a=0$ and $F_{ext}-T-\mu m_2g=m_2a=0$. Adding these gives $F_{ext}=\mu(m_1+m_2)g$ or $\mu= F_{ext}/(m_1+m_2)g$=24.0N/(22.0kg x 9.8m/s^2) = 0.111.

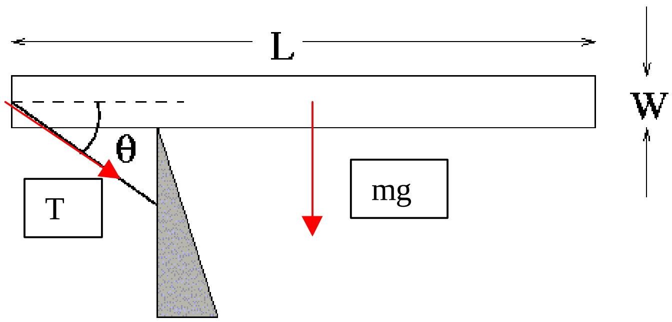
3. **(20 points)** A duck (mass M=1.00 kg) flying **east** at 30.0 m/s is killed by an arrow of mass m=0.200 kg. After the hit, the duck and arrow are travelling **60^o north of east** at 50.0 m/s.

   a. **(10 points)** What direction was the arrow going just before the hit? Ans: The momentum of the duck just before the hit was p_{duck}= 30.0 kgm/s **east**. The momentum after the collision was the total mass, 1.20 kg, times the final velocity, 50.0 m/s **60^o north of east**, or 60.0 kgm/s **60^o north of east**. The momentum of the arrow is the difference between these vectors, as shown below, and points **north.**

   b. **(10 points)** What was the speed of the arrow just before the hit? Ans: The momentum of the arrow had magnitude (60.0 kgm/s)sin(60^o)=52.0 kgm/s. Dividing by the mass 0.200 kg of the arrow gives the speed, 260. m/s.

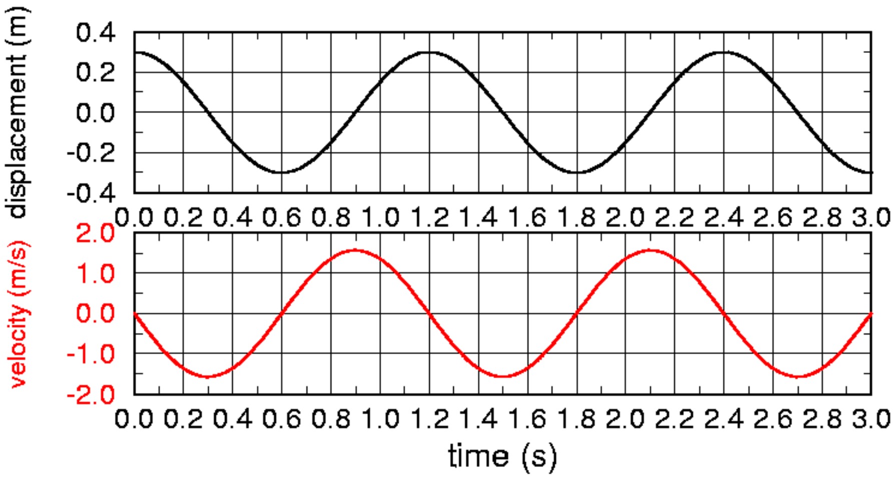


4. (20 points -show your work.) Shown below is a boom of length L=4.00 m and width W negligible. It is pivoted on a firm support at a distance 0.80 m from the end. The boom is uniform and has mass m=50.0 kg. A cable making an angle q=30.0^{o} is attached to the left end and keeps the system in equilibrium. The moment of inertia is $I=52mL^2/300$ for a uniform bar around a point one fifth of the way from an end.



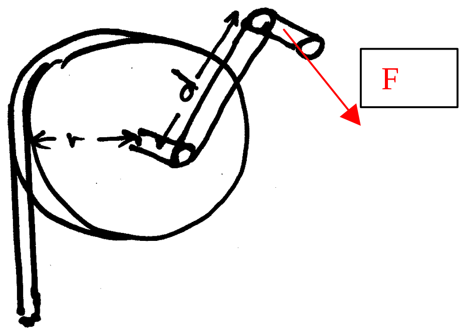
- **a. (7 points)** How large is the tension T in the cable? Ans: Calculate torques relative to the pivot point where the support touches the boom. The torque due to the weight of the boom is 3mgL/10 clockwise. The torque due to the tension T in the cable is T(2L/10)sin(30.0^{o}) counterclockwise. These balance, so T=3mg/ 2sin(30.0^{o})=1470 N.
- **b. (8 points)** Suddenly the cable snaps. What is the angular acceleration around the pivot point? Ans: The torque due to gravity, 3mgL/10, now equals $I\alpha=(52/300)mL^2\alpha$. Therefore $\alpha=(90/52)g/L=4.24\ s^{-2}$
- **c. (5 points)** What is the linear acceleration of the right hand end of the boom? Ans: $a=R\alpha$, where the radius arm is 4L/5=3.2**m**, so a=13.6m/s^2. It is interesting that this exceeds 9.8m/s^2, the acceleration in free fall of a point particle.

5. (**20 points -show your work unless the answer requires only simple inspection.**)
The graph below shows a space-time history of a particle of mass 0.250 kg oscillating on a spring



a. (**3 points**) What is the frequency in Hertz? Ans: f=1/period=1/1.20s=0.833 Hz
b. (**3 points**) What is the velocity at *t*=0.30 s? Ans: The velocity is the slope of displacement versus time. The point t=0.30 s is the moment where velocity has its largest negative value, -2πfA, with A=0.30m the amplitude. Thus v = -1.57 m/s.
c. (**2 points**) What is the acceleration at *t*=0.30 s? Ans: The spring is not stretched so the force and acceleration are both 0.
d. (**2 points**) What is the velocity at *t*=0.60 s? Ans: The slope is 0 so v=0.
e. (**2 points**) What is the force at *t*=0.9 s? Ans: The spring is not stretched so F=0.
f. **(2 points)** What is the spring constant? Ans: $k=m(2\pi f)^2=6.85$ N/m.
g. (**3 points**) How much energy does the oscillator have? Ans: $E=\frac{1}{2}kA^2=0.308$ J.
h. **(3 points)** on the blank graph provided, draw the graph of *v(t)*, the velocity of the oscillator. Be sure to label and indicate the scale on the axis.

6. (20 points) A cylindrical wheel (mass m=1.20 kg, radius r=0.30 m, moment of inertia $I=mr^2/2$) is turned using a crank of negligible mass and length d=0.50 m. The angular acceleration a is 1.6 rad/s^2. Friction can be neglected.



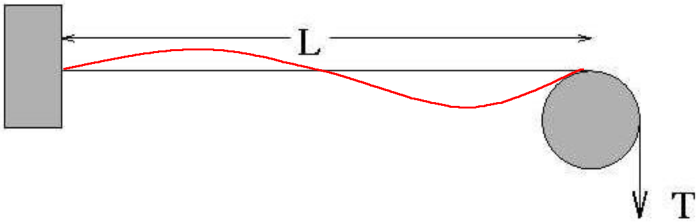
(a) **(5 points)** What is the minimum force F which must be applied to the crank? Ans: The force F should be applied perpendicular to the arm of length d so that the torque around the axis is Fd. By Newton's law this is $I\alpha=(mr^2/2)\alpha$. Thus the force is $F= (mr^2/2)\alpha/d=0.173$ N.

(b) **(5 points)** If there is a massless rope attached as shown, and if the system starts from rest, how long does it take for 15.0 m of rope to be wrapped? Ans: The acceleration at the rim is $a= r\alpha=0.48$ m/s^2. The time is found from 15m= $\frac{1}{2}$ at^2. Therefore t=7.91 s.

(c) **(5 points)** Suppose a bucket of mass M=3.5 kg is attached to the rope, and the angular acceleration a is still 1.6 rad/s^2. What is the tension in the rope? Ans: Newton's law gives T-Mg=Ma for the bucket, where a is still 0.48 m/s^2. Therefore T=M(g+a)=36.0 N.

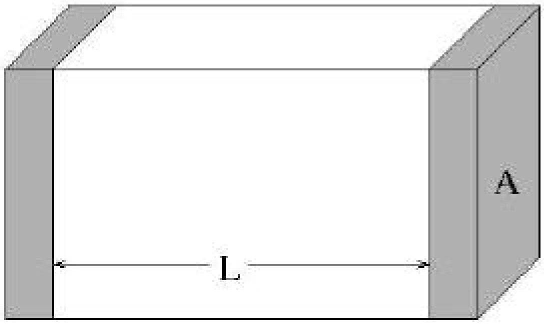
(d) **(5 points)** What is now the minimum force F? Ans: The total torque on the cylindrical wheel is Fd-Tr and this must equal $I\alpha$. Thus the force is $I\alpha/d + Tr/d$. The first term is just the previous force, 0.173 N, and the second is 21.59 N, giving F=21.8 N.

7. **(20 points)** The string shown below has mass m=0.0150 kg and length L=1.80 m. A standing wave, the 2nd harmonic (or 1st overtone) is excited with amplitude A=0.0250 m at a frequency f= 30.0 Hz.



a. **(4 points)** What is the wavelength? Ans: λ=L=1.80 m
b. **(4 points)** What is the wave velocity of the string? Ans: $v_w = \lambda f$=54.0 m/s.
c. **(4 points)** What is the tension T on the string? Ans: $T=\mu v_w^{\ 2}$ where μ is m/L, the mass per unit length. Thus T=24.3 N.
d. **(4 points)** What is the maximum displacement of a piece of string at a distance 0.30 m from the left end? Ans: The standing wave has the formula $d(x,t)= A \sin(2\pi x/\lambda)\sin(2\pi ft)$. The maximum is $A \sin(2\pi x/\lambda)$=(0.0250 m)sin(1.047 rad) =0.0217 m.
e. **(4 points)** What is the maximum velocity of a piece of string at this position? Ans: The maximum velocity is $2\pi f$ times the maximum displacement, or 4.08 m/s.

8. **(20 points)** The vertical parallel walls shown below have area A=1.200m^2 and are separated by a distance L=0.400m. The left wall is held at temperature T=40.0^{o}C and the right wall at temperature T=10^{o}C. How much heat is transferred in an hour if:



a. **(10 points)** The space is filled with cork with thermal conductivity κ=0.040 W/mK. Ans: $Q=(\Delta Q/\Delta t)t$ and $\Delta Q/\Delta t=-\kappa A \Delta T/L$. Thus Q=(3600s)(0.040 W/mK)(1.200m^2)(30 K)/(0.400 m)=1.30 x 10^4 J.
b. **(10 points)** The space is empty ("filled" with vacuum.) The walls have emissivity e=0.45. Ans: $Q=(\Delta Q/\Delta t)t$ and $\Delta Q/\Delta t=e\sigma A(T_1^{\,4}-T_2^{\,4})$. Therefore Q=(3600 s)(0.45)(5.67 x 10^{-8} W/m^{2}K^4)(1.200m^2)(313^4 K^4-283^4 K^4)=3.51 x10^5 J.

9. **(20 points)** 1.20 kg of water at 100^oC and at one atmosphere pressure is turned to steam at 100^oC and at one atmosphere pressure. (H_2O has atomic mass 18.)

   a. **(4 points)** How much heat was added? Ans: $Q=mL_v=2.707 \times 10^6$ J.
   b. **(4 points)** How much ice (at 0^oC) would this amount of heat melt (to water at 0^oC)? Ans: $m' = mL_v / L_f = 8.11$ kg.
   c. **(4 points)** What is the volume of the resulting steam? Ans: $V=nRT/P$ where the number of moles $n=m/M=1200g/(18g/mole)=66.7$ mole. Therefore $V= (66.7 \text{ mole})(8.315 \text{ J/moleK})(373 \text{ K})/(1.013 \times 10^5 \text{ N/m}^2)=2.04 \text{ m}^3$.
   d. **(4 points)** How much work was done by the "system" (that is, the H_2O)? Ans: $W=P\Delta V=(1.013 \times 10^5 \text{ N/m}^2)(2.04 \text{ m}^3 - 0.0012 \text{ m}^3)=2.07 \times 10^5$ J.
   e. **(4 points)** How much did the internal energy *U* of the H_2O increase? Ans: $\Delta U= Q-W=2.707 \times 10^6 \text{ J} - 0.207 \times 10^6 \text{ J}=2.50 \times 10^6$ J.

10. **(20 Points)** One (1.00) kg of (monatomic) neon (Ne, atomic mass M=20.2) is in a box of volume V_1=0.800 m^3. It is an ideal gas to good approximation. The temperature is 20.0°C.

a. **(4 Points)** How many atoms are there? Ans: n=m/M=1000 g/(20.2 g/mole)=49.5 moles; $N=nN_A$ =2.98 x 10^{25} atoms.

b. **(4 Points)** What is their total kinetic energy? Ans: = $\frac{3}{2}$nRT= $\frac{3}{2}$$Nk_BT$= 1.81 x 10^5 J.

c. **(3 Points)** The gas expands into a volume V_2=1.200 m^3 without changing the temperature. How much work was done by the gas? Ans: W= $\int$ PdV=nRTln(V_2/V_1)=(49.5 moles)(8.315 J/moleK)(293 K)ln(1.5)=4.89 x 10^4 J.

d. **(3 Points)** How much heat was absorbed by the gas? Ans: ΔU=0 so Q=W (same as c.)

e. **(3 Points)** The gas now expands adiabatically to a volume V_3=1.600 m^3. What is the temperature? Ans: the specific heat ratio γ=5/3 and $TV^{\gamma-1}$=const. Therefore $T_3=T_2(V_2/V_3)^{2/3}$=(293K)$(.75)^{2/3}$=242K= -31°C.

f. **(3 Points)** How much heat was absorbed during the expansion from V_2 to V_3? Ans: no heat (adiabatic!)