Physics 131 Midterm Number 2 Answers to Version 2 (4/13/01)

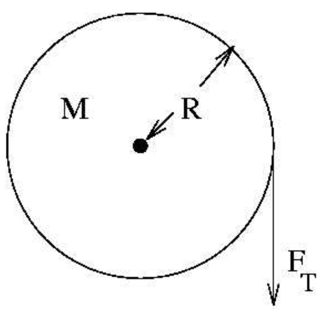
1. **(20 points)** An arrow (mass m=0.29 kg) travelling north at 150 m/s kills a duck (mass M=2.0 kg) flying east at 29 m/s.
   a. **(8 points)** At what angle (north of east) does the dead duck (plus arrow) travel immediately after the hit?
   b. **(7 points)** What is the speed of the duck and arrow just after the hit?
   c. **(5 points)** How much mechanical energy change occurs during the hit?

a. The arrow has momentum mv=(0.29 kg)(150 m/s)=43.5 kgm/s to the East. The duck has momentum MV=(2.0 kg)(29 m/s)=58 kgm/s to the North. After the hit, the total vector momentum stays the same, and has components (43.5,58) kgm/s (East, North).
The angle is $\tan^{-1}(58/43.5)=53.^{o}$ N of E.
b. The speed is the magnitude of the momentum divided by the total mass, or

$$\sqrt{(43.5)^2+(58)^2}\ /(.29+2.0)=32.\text{ m/s}$$

c. Before the hit, the arrow has kinetic energy $mv^2/2$=3260 J, and the duck has kinetic energy $MV^2/2$= 841J. The total energy was 4100J. After the hit, the kinetic energy was $(m+M)v'^2/2$= 1170 J. The energy lost is 2900 J.

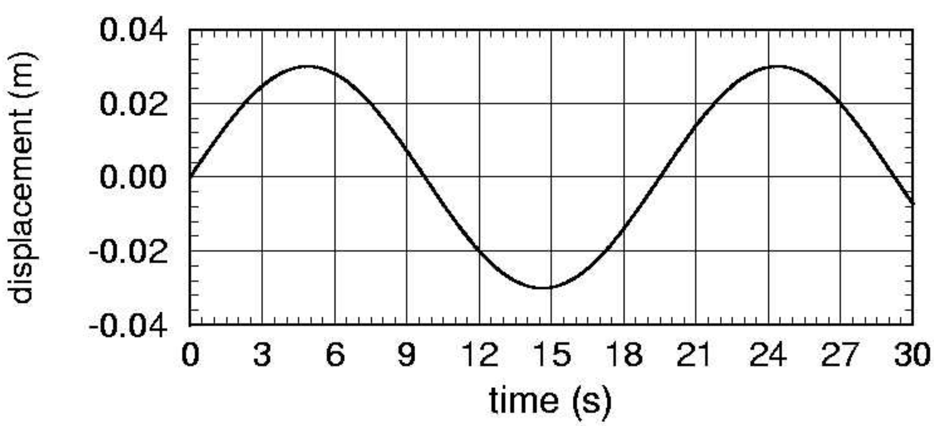
2. (**20 points – show your work.**) The cylinder shown below, pivoted on an axis through the center, has mass M=30.0 kg and radius R=0.11 m. The formula for the moment of inertia of a cylinder is $I=MR^2/2$.



a. (**5 points**) A force F_T is applied as shown, using a massless rope wrapped around the cylinder. How big should F_T be to get an angular acceleration α= 5.5 rad/s^2?
b. (**5 points**) The cylinder is initially at rest. For how long should the force F_T act in order to obtain a final frequency of rotation f=0.80 revolutions/s?
c. (**5 points**) After achieving f=0.80 revolutions/s, the force F_T is removed and frictional forces cause the cylinder to come to rest in 25 s. How much work did friction do?
d. (**5 points**) During the time when the force F_T was accelerating the cylinder from f=0 to f=0.80 revolutions/s, how far was the rope pulled?

a. The torque is $F_T R = I\alpha$. Using I=0.181 kgm^2, we get $F_T = I\alpha/R$ = 9.08 N.
b. The final angular velocity is $\omega=\alpha t$ and $\omega=2\pi f$. Therefore $t=2\pi f/\alpha$ = 0.91 s.
c. The work done by friction is the change in kinetic energy. The final kinetic energy is 0 and the initial kinetic energy is $I\omega^2/2$ = 2.29 J. Therefore the work done by friction is –2.29 J.
d. The total work done by the rope is $F_T y$ where y is the distance the rope was pulled. The total work done by the rope is also 2.29 J since this is the kinetic energy that the cylinder gained. Therefore $y=W/F_T$ = 0.25 m.

3. (**20 points – show your work unless the answer requires only simple inspection.**) A mass M=0.25 kg oscillates without friction as shown in the graph below.
   a. (**3 points**) What is the amplitude of oscillation?
   b. (**4 points**) What is the angular frequency ω of oscillation?
   c. (**3 points**) What is the phase ϕ of this oscillator using the convention $x=A\cos(\omega t+\phi)$, where x is the displacement?
   d. (**4 points**) What is the maximum velocity of the mass?
   e. (**4 points**) What is the velocity at time t=11 s?
   f. (**4 points**) How much energy does the oscillator have?



a. By inspection the amplitude A =0.030 m.
b. ω is $2\pi/T$ and the period T is 19.5 s. Therefore ω= 0.32 rad/s.
c. The phase is $3\pi/2$ or $-\pi/2$ (they are equivalent and both correct.)
d. The maximum velocity is ωA =0.0097 m/s.
e. At t=11s we have $x=A\cos(\omega t+\phi)$ so $v=-\omega A\sin(\omega t+\phi)$ with t = 11 s, or (-0.0097 m/s)sin(3.52-1.57)= -0.0090 m/s. This can be checked by looking at the graph and estimating the slope.
f. The most convenient formula is $mv_{max}^2/2$= 1.18 x 10^{-5} J.