1. **(20 points)** An arrow (mass \(m=0.29 \text{ kg}\)) travelling north at 150 m/s kills a duck (mass \(M=2.0 \text{ kg}\)) flying east at 29 m/s.
   a. **(8 points)** At what angle (north of east) does the dead duck (plus arrow) travel immediately after the hit?
   b. **(7 points)** What is the speed of the duck and arrow just after the hit?
   c. **(5 points)** How much mechanical energy change occurs during the hit?

   a. The arrow has momentum \(mv=(0.29 \text{ kg})(150 \text{ m/s})=43.5 \text{ kgm/s}\) to the East. The duck has momentum \(MV=(2.0 \text{ kg})(29 \text{ m/s})=58 \text{ kgm/s}\) to the North. After the hit, the total vector momentum stays the same, and has components \((43.5,58) \text{ kgm/s}\) (East, North). The angle is \(\tan^{-1}(58/43.5)=53.0^\circ\) N of E.
   b. The speed is the magnitude of the momentum divided by the total mass, or
   
   \[
   \sqrt{(43.5)^2+(58)^2}/(0.29+2.0)=32. \text{ m/s}
   \]
   c. Before the hit, the arrow has kinetic energy \(mv^2/2=3260 \text{ J}\), and the duck has kinetic energy \(MV^2/2=841\text{ J}\). The total energy was 4100J. After the hit, the kinetic energy was \((m+M)v'2/2=1170 \text{ J}\). The energy lost is 2900 J.

2. **(20 points – show your work.)** The cylinder shown below, pivoted on an axis through the center, has mass \(M=30.0 \text{ kg}\) and radius \(R=0.11 \text{ m}\). The formula for the moment of inertia of a cylinder is \(I=MR^2/2\).

![Diagram of a cylinder with a force \(F_T\) applied](image)

   a. **(5 points)** A force \(F_T\) is applied as shown, using a massless rope wrapped around the cylinder. How big should \(F_T\) be to get an angular acceleration \(\alpha=5.5 \text{ rad/s}^2\)?
   b. **(5 points)** The cylinder is initially at rest. For how long should the force \(F_T\) act in order to obtain a final frequency of rotation \(f=0.80 \text{ revolutions/s}\)?
   c. **(5 points)** After achieving \(f=0.80 \text{ revolutions/s}\), the force \(F_T\) is removed and frictional forces cause the cylinder to come to rest in 25 s. How much work did friction do?
   d. **(5 points)** During the time when the force \(F_T\) was accelerating the cylinder from \(f=0\) to \(f=0.80 \text{ revolutions/s}\), how far was the rope pulled?
a. The torque is \( F_R R = I \alpha \). Using \( I = 0.181 \text{ kgm}^2 \), we get \( F_R = I \alpha / R = 9.08 \text{ N} \).

b. The final angular velocity is \( \omega = \alpha t \) and \( \omega = 2\pi f \). Therefore \( t = 2\pi / \alpha = 0.91 \text{ s} \).

c. The work done by friction is the change in kinetic energy. The final kinetic energy is 0 and the initial kinetic energy is \( I \omega^2 / 2 = 2.29 \text{ J} \). Therefore the work done by friction is \(-2.29 \text{ J} \).

d. The total work done by the rope is \( F_T y \) where \( y \) is the distance the rope was pulled. The total work done by the rope is also 2.29 J since this is the kinetic energy that the cylinder gained. Therefore \( y = W / F_T = 0.25 \text{ m} \).

3. (20 points – show your work unless the answer requires only simple inspection.) A mass \( M = 0.25 \text{ kg} \) oscillates without friction as shown in the graph below.

a. (3 points) What is the amplitude of oscillation?

b. (4 points) What is the angular frequency \( \omega \) of oscillation?

c. (3 points) What is the phase \( \phi \) of this oscillator using the convention \( x = A \cos(\omega t + \phi) \), where \( x \) is the displacement?

d. (4 points) What is the maximum velocity of the mass?

e. (4 points) What is the velocity at time \( t = 11 \text{ s} \)?

f. (4 points) How much energy does the oscillator have?

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[Graph of oscillation showing time on the x-axis and displacement on the y-axis.]

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a. By inspection the amplitude \( A = 0.030 \text{ m} \).

b. \( \omega \) is \( 2\pi / T \) and the period \( T \) is 19.5 s. Therefore \( \omega = 0.32 \text{ rad/s} \).

c. The phase is \( 3\pi / 2 \) or \(-\pi / 2 \) (they are equivalent and both correct.)

d. The maximum velocity is \( \omega A = 0.0097 \text{ m/s} \).

e. At \( t = 11 \text{ s} \) we have \( x = A \cos(\omega t + \phi) \) so \( v = -\omega A \sin(\omega t + \phi) \) with \( t = 11 \text{ s} \), or \( (0.0097 \text{ m/s}) \sin(3.52 - 1.57) = -0.0090 \text{ m/s} \). This can be checked by looking at the graph and estimating the slope.

f. The most convenient formula is \( mv_{\text{max}}^2 / 2 = 1.18 \times 10^{-5} \text{ J} \).