12.50 An experiment is performed in deep space with two uniform spheres, one with mass 25.0 kg and the other with mass 100.0 kg. They have equal radii, $r = 0.20$ m. The spheres are released from rest with their centers 40.0 m apart. They accelerate toward each other because of their mutual gravitational attraction. You can ignore all gravitational forces other than that between the two spheres. a) Explain why linear momentum is conserved. b) When their centers are 20.0 m apart, find i) the speed of each sphere and ii) the magnitude of the relative velocity with which one sphere is approaching the other. c) How far from the initial position of the center of the 25.0-kg sphere do the surfaces of the two spheres collide?

12.50: Denote the 25-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2. a) Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1v_1 = m_2v_2$. This relationship is useful in solving part (b) of this problem. b) From the work-energy theorem,

$$Gm_1m_2\left[\frac{1}{r_f}-\frac{1}{r_i}\right]=\frac{1}{2}\left(m_1v_1^2+m_2v_2^2\right)$$

and from conservation of momentum the speeds are related by $m_1v_1 = m_2v_2$. Using the conservation of momentum relation to eliminate v_2 in favor of v_1 and simplifying yields

$$v_1^2=\frac{2Gm_2^2}{m_1+m_2}\left[\frac{1}{r_f}-\frac{1}{r_i}\right],$$

with a similar expression for v_2. Substitution of numerical values gives $v_1 = 1.63\times10^{-5}$ m/s, $v_2 = 4.08\times10^{-6}$ m/s. The magnitude of the relative velocity is the sum of the speeds, 2.04×10^{-5} m/s.

c) The distances the centers of the spheres travel (x_1 and x_2) is proportional to their acceleration, and $\frac{x_1}{x_2}=\frac{a_1}{a_2}=\frac{m_2}{m_1}$, or $x_1 = 4x_2$. When the spheres finally make contact, their centers will be a distance of $2R$ apart, or $x_1 + x_2 + 2R = 40$ m, or $x_2 + 4x_2 + 2R = 40$ m. Thus, $x_2 = 8\text{ m} - 0.4R$, and $x_1 = 32\text{ m} - 1.6R$.

13.24 A 0.500-kg glider, attached to the end of an ideal spring with force constant $k = 450$ N/m, undergoes simple harmonic motion with an amplitude 0.040 m. Compute a) the maximum speed of the glider; b) the speed of the glider when it is at $x = -0.015$ m; c) the magnitude of the maximum acceleration of the glider; d) the acceleration of the glider at $x = -0.015$ m; e) the total mechanical energy of the glider at any point in its motion.

13.24: a) From Eq. (13.23),

$$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{450\ \text{N/m}}{0.500\ \text{kg}}}(0.040\ \text{m}) = 1.20\ \text{m/s}.$$

b) From Eq. (13.22),

$$v = \sqrt{\frac{450\ \text{N}}{0.500\ \text{kg}}}\sqrt{(0.040\ \text{m})^2 - (-0.015\ \text{m})^2} = 1.11\ \text{m/s}.$$

c) The extremes of acceleration occur at the extremes of motion, when $x = \pm A$, and

$$a_{max} = \frac{kA}{m} = \frac{(450\ \text{N/m})(0.040\ \text{m})}{(0.500\ \text{kg})} = 36\ \text{m/s}^2$$

d) From Eq. (13.4), $a_x = -\frac{(450\ \text{N/m})(-0.015\ \text{m})}{(0.500\ \text{kg})} = 13.5\ \ \text{m/s}^2$.

e) From Eq. (13.31), $E = \frac{1}{2}(450\ \text{N/m})(0.040\ \text{m})^2 = 0.36\ \text{J}$.