

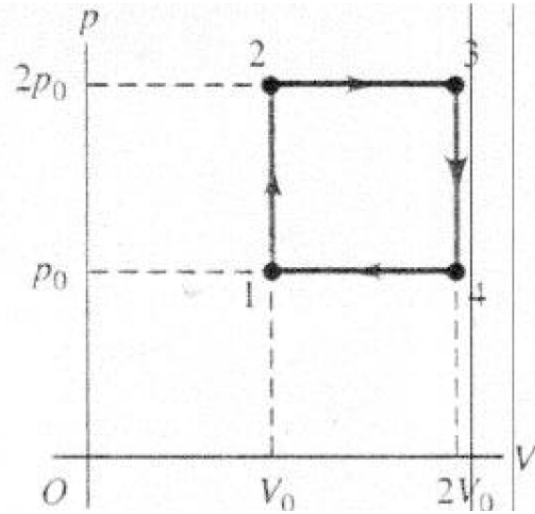
**Phy131 Spring 2006. Practice problems from Ch. 19 & 20** – I didn't have time to explain these enough in lecture.

**19.59** An air pump has a cylinder 0.250 m long with a movable piston. The pump is used to compress air from the atmosphere (at absolute pressure  $1.01 \times 10^5$  Pa) into a very large tank at  $4.20 \times 10^5$  Pa gauge pressure. (For air,  $C_V = 20.8$  J/mol·K.)

a) The piston begins the compression stroke at the open end of the cylinder. How far down the length of the cylinder has the piston moved when air first begins to flow from the cylinder into the tank? Assume that the compression is adiabatic. b) If the air is taken into the pump at  $27.0^\circ\text{C}$ , what is the temperature of the compressed air? c) How much work does the pump do in putting 20.0 mol of air into the tank?

- a) Adiabatic compression of an ideal gas says that  $p_0 L_0^\gamma = p L^\gamma$  where the volume  $V$  is proportional to the length  $L$  of the cylindrical chamber whose length is  $L_0$  when the piston is all the way out, and 0 when the piston is all the way in. The constant  $\gamma=1.4$  for a diatomic gas like air. The value of  $L$  is found by choosing  $p$  to be  $(4.20 + 1.01) \times 10^5$  Pa, and  $p_0$  to be one atmosphere,  $1.01 \times 10^5$  Pa. The result is that  $L/L_0 = (p_0/p)^{1/\gamma} = 0.309$ , or the piston starts pushing air out of the pump into the tank when the piston has been pushed by  $L_0 - L = 0.173$  m.
- b) At this pressure, the temperature can be found from  $T_0 L_0^{\gamma-1} = T L^{\gamma-1}$ . Remember that  $T$  and  $T_0$  are in absolute units, so  $T_0 = 273 + 27$  K = 300K. Then  $T = T_0 (L_0/L)^{0.4} = 480$  K =  $207^\circ\text{C}$ .
- c) The work  $W_{ad}$  needed to compress adiabatically is  $-\Delta U$  since  $\Delta U = Q - W$  and  $Q=0$  in an adiabatic process. For an ideal gas,  $\Delta U = nC_V \Delta T$  and  $C_V = (5/2)R$  for a diatomic gas. This gives  $W = 7.5 \times 10^4$  J. An alternate calculation uses eq. 19.26 which is  $W = (p_1 V_1 - p_2 V_2) / (\gamma - 1) = nR(T_1 - T_2) / (\gamma - 1)$  and gives the same answer. There is additional work pushing the gas into the tank at the constant pressure  $p$  of the tank. This work is just  $pV$  where  $V$  is the total volume pushed in at pressure  $p$ . Therefore we can use instead  $nRT$ , where  $T$  is the absolute temperature, 480K. The result is an additional  $8.0 \times 10^4$  J.

**20.45** What is the thermal efficiency of an engine that operates by taking  $n$  moles of diatomic ideal gas through the cycle  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$  shown in Fig. 20.22?



Denote the temperature at point 1 by  $T_0$  (that is  $p_0V_0=nRT_0$ ). Then the temperatures at points 2 and 4 are  $2T_0$  and at point 3 it is  $4T_0$ . A Carnot engine operating between  $T_0$  and  $4T_0$  has efficiency  $e=1-|Q_c/Q_h|=1-1/4=3/4$ . This engine must be less efficient, and in fact, is much less, with  $e=2/19$ . The proof is as follows. The general definition of efficiency is  $e=W/Q_{in}$ , the ratio of work (area of the loop,  $p_0V_0$  in this case) to the heat that enters in steps  $1 \rightarrow 2$  and  $2 \rightarrow 3$  (heat leaves in the other two steps.) At constant volume,  $Q_{12}=nC_v\Delta T$  and at constant pressure,  $Q_{23}=nC_p\Delta T$ . Thus we have  $Q_{in}=n((5/2)R(T_0)+(7/2)R(2T_0))$ , using the formulas for a diatomic gas, and the values of  $\Delta T$  for the two processes. An alternate method to find  $Q_{12}+Q_{23}$  is to use  $Q=\Delta U+W$  where  $\Delta U$  is the total energy change going from 1 to 3, which is just  $nC_v(3T_0)$ , and where  $W$  is the work done in step  $2 \rightarrow 3$ , namely  $2p_0V_0$ . Both methods give  $Q=(19/2)p_0V_0$ .

**20.27** Calculate the entropy change that occurs when 1.00 kg of water at  $20.0^\circ\text{C}$  is mixed with 2.00 kg of water at  $80.0^\circ\text{C}$ .

Entropy ( $S$ ) is found using the primary facts (1) that entropy depends only on the equilibrium state that the system is in, not the path by which it got there, and (2) that in a reversible isothermal process at temperature  $T$ , with heat input  $\Delta Q$ ,  $\Delta S = \Delta Q/T$ . When 2 kg of water at  $80^\circ\text{C}$  mixes with 1 kg at  $20^\circ\text{C}$ , the 2 kg part cools from  $T_h = 273+80\text{K}$  to  $T_f = 273+60\text{K}$ , while the 1 kg part heats from  $T_c = 273 + 20\text{K}$  to  $T_f = 273 + 60\text{K}$ . The process of cooling involves decreasing entropy, and the process of heating involves increasing entropy. The net entropy change must be positive (for most, irreversible, process, like this mixing process) or zero (for a reversible process), never negative.

The processes which actually happen in nature are irreversible. However, the  $\Delta S$  values can be computed for an equilibrium process which connects the same initial and final states. So we imagine a sequence of additions or subtractions of heat, each happening at approximately a constant temperature  $T_i$ , with  $T_i$  changing slowly until we reach the final temperature. Thus we have  $\Delta S = \sum \Delta Q_i/T_i$  which becomes the integral

$$\Delta S = \int dQ/T = \int \frac{nC}{T} dT = nC \ln(T_f/T_i), \text{ where } T_i \text{ and } T_f \text{ are initial and final}$$

temperatures, and  $C$  is  $C_V$  or  $C_p$  for constant volume or constant pressure processes. In our case, the medium is liquid water, which does not have a large thermal expansion, so  $C_V$  and  $C_p$  are practically the same. Adding the entropy changes for the heating and the cooling, we have  $\Delta S = (1\text{kg})c \ln(333/293) + (2\text{kg})c \ln(333/353)$ , using  $mc$  in place of  $nC$ . This gives  $\Delta S = (4190\text{J/K})(0.12797-0.11665)=47.4\text{ J/K}$ , a positive number, as always happens when heat flows from hot to cold irreversibly.