**Physics 131 Spring 2006.  Formulas you should memorize for Midterm #2** (of course, you must also know the formulas for Midterm #1.)

<table>
<thead>
<tr>
<th>Angular Kinematics</th>
<th>( \theta \equiv s(=\text{arc length})/R = \theta(t) ) (angular position)</th>
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</thead>
<tbody>
<tr>
<td>rotation angle ( \theta ) (radians; rotation radius ( R )):</td>
<td>( \omega \equiv v/\tan R; \ \alpha \equiv a/\tan \ ) \tan = \text{tangential}</td>
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<tr>
<td>angular velocity ( \omega ); angular acceleration ( \alpha )</td>
<td>( \omega = \omega_0 + \alpha t; \ \theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 )</td>
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<tr>
<td>Circular motion (radius ( R )) with constant ( \alpha ):</td>
<td>eliminating ( t ): ( \omega = \omega_0 + 2\alpha(\theta - \theta_0) )</td>
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<td>circular motion – radial acceleration ( a_{\text{rad}} ):</td>
<td>( a_{\text{rad}} = a = v^2/T ) (radially inwards)</td>
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<td>Center-of-Mass position of system (mass ( M ))</td>
<td>( r_{\text{cm}} \equiv \sum m_i r_i / \sum m_i = \sum m_i r_i / M )</td>
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<tr>
<td>Moment of Inertia ( I ):</td>
<td>( I = \sum m_i r_i^2 = \int r^2 dm; r_i(r) = \text{distance between rotation axis and } m_i(dm) )</td>
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<tr>
<td>I for thin hollow cylinder, about the central symmetry axis</td>
<td>( I = MR^2 ) More complicated formulas in table 9.2 do <strong>NOT</strong> need to be memorized. They will be given if needed.</td>
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<tr>
<td>Parallel-Axis Theorem</td>
<td>( I = I_{/i,\text{cm}} + Md^2 ) (( d=\text{distance between the parallel axes} ))</td>
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<tr>
<td>Momentum ( p )</td>
<td>( p \equiv \sum m_i v_i = Mv_{\text{cm}} )</td>
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<tr>
<td>Angular Momentum ( L )</td>
<td>( L = \sum R_i \times m_i v_i ) (= ( I_0 \omega ) for a rigid body rotating around the axis used when computing ( I ))</td>
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<tr>
<td>Kinetic Energy ( K ) ([\text{J} \equiv \text{Nm}]):</td>
<td>Moving axis: ( K_{\text{tot}} \equiv \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega_{\text{cm}}^2 )</td>
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<tr>
<td>Fixed axis A: ( K_{\text{rot,A}} \equiv \frac{1}{2} I_{\text{A}} \omega_{\text{A}}^2 )</td>
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<tr>
<td>Forces ([\text{N} \equiv \text{kgm/s}^2] )</td>
<td>( \sum F_i = ma = dp/dt; \ F_{A \text{on } B} = -F_{B \text{on } A}; \ \sum \tau = dL/dt = l\alpha )</td>
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<tr>
<td>Torques ([\text{Nm or J}] ) and consequences:</td>
<td>In PHY131, rotations will always be in a fixed plane. The vectors ( \tau, \ \omega, ) etc. will therefore have a fixed direction perpendicular to the plane, and you do not need to worry about direction, just <strong>sign</strong>. Conventionally, counterclockwise is positive, but you may <strong>specify</strong> your own convention.</td>
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<tr>
<td>Force of Gravity between $M$ and $m$, at center-to-center distance $r$</td>
<td>$F_G = \frac{GMm}{r^2}$ (attractive! $G=6.67\times10^{-11}$ Nm$^2$/kg$^2$); near sea level: $F_G = mg(-j)$ (downwards; $g=9.80$ m/s$^2$). Memorize 9.80, but not the numerical value $6.67\times10^{-11}$ Nm$^2$/kg$^2$</td>
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<td>Force of a Spring (spring constant $k$):</td>
<td>$F_S = -kx$ (opposes compression/stretch)</td>
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<td>Torque:</td>
<td>$\tau \equiv \mathbf{R} \times \mathbf{F}$ ($\tau = RF\sin\theta_{RF}$; direction: right-hand rule)</td>
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<td>Equilibrium &amp; Collisions:</td>
<td>$\Sigma F_i = \frac{dp_{\text{tot}}}{dt} = 0$ and $\Sigma \tau_i = \frac{dL_{\text{tot}}}{dt} = 0 \Rightarrow \Delta p_{\text{tot}} = \Delta L_{\text{tot}} = 0$ Elastic: $K$ is conserved; Completely Inelastic: objects stick together afterwards</td>
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<tr>
<td>Work done by a torque $\tau_F$ over a rotation by angle $\theta$:</td>
<td>$W_F \equiv \int \tau_F \cdot d\theta$</td>
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<td>Power $P$ [W≡J/s]:</td>
<td>$P_F \equiv dW_F/dt = \mathbf{F} \cdot \mathbf{v}$ or $\tau_F \cdot \omega$</td>
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<tr>
<td>Potential Energy $U$ of conservative forces $F$</td>
<td>$U_F = -W_F$; e.g. $U_G = -\frac{GMm}{r}$, $U_G$ (near earth)$=mgy$; $U_s = \frac{1}{2}kx^2$</td>
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<tr>
<td>Harmonic oscillator</td>
<td>$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$ The “circle of reference” is a picture where the oscillator displacement $x$ is horizontal, while $-v/\omega$ is vertical. The oscillator is then “visualized” as a point on a circle of radius $A$, whose angular motion $\theta(t)$ is given by $\theta=\omega t+\phi$, and $x(t)=A\cos\theta$.</td>
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<td>$x(t) = A\cos(\omega t + \phi)$ where $\omega = \sqrt{k/m}$ if the damping coefficient $b=0$. $\omega = 2\pi f=2\pi/T$. $A = \sqrt{x_0^2+(v_0/\omega)^2}$; $\phi = -\tan^{-1}(v_0/\omega x_0)$ when $b \neq 0$ the oscillations die exponentially as $e^{-\gamma t}$ where you do not need to memorize formulas like $\gamma = b/2m$ or the formula for the shifted resonant frequency.</td>
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<td>Pendulum</td>
<td>$I \frac{d^2\theta}{dt^2} = -MgL_{\text{cm}}\sin \theta = -MgL_{\text{cm}}\theta$ The small angle approximation is used in the second formula.</td>
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<td>$\theta(t) = \Theta \cos(\omega t + \phi)$ where $\omega = \sqrt{MgL_{\text{cm}}/I}$ for the general pendulum, which becomes $\omega = \sqrt{g/L}$ for the simple pendulum. These results depend on the angular amplitude $\Theta$ being small.</td>
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