

**Physics 131 Spring 2006. Formulas you should memorize for Midterm #2** (of course, you must also know the formulas for Midterm #1.)

<b>Angular Kinematics</b> rotation angle $\theta$ (radians; rotation radius $R$ ):	$\theta \equiv s(\text{=arc length})/R = \theta(t)$ (angular position)
angular velocity $\omega$ ; angular acceleration $\alpha$	$\omega \equiv v_{\text{tan}}/R$ ; $\alpha \equiv a_{\text{tan}}/R$ ( $\text{tan} = \text{tangential}$ )
Circular motion (radius $R$ ) with <b>constant</b> $\alpha$ :  circular motion – radial acceleration $a_{\text{rad}}$ :	$\omega = \omega_0 + \alpha t$ ; $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ ; eliminating $t$ : $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $a_{\text{rad}} = a_c = v_T^2/R$ (radially inwards)
Center-of-Mass position of system (mass $M$ )	$\mathbf{r}_{\text{cm}} \equiv \sum_i m_i \mathbf{r}_i / \sum_i m_i = \sum_i m_i \mathbf{r}_i / M$
Moment of Inertia $I$ :	$I \equiv \sum_i m_i r_i^2 = \int r^2 dm$ ; $r_i(r)$ = distance between rotation axis and $m_i(dm)$
$I$ for thin hollow cylinder, about the central symmetry axis	$I = MR^2$ More complicated formulas in table 9.2 do <b>NOT</b> need to be memorized. They will be given if needed.
Parallel-Axis Theorem	$I = I_{\text{cm}} + Md^2$ ( $d$ =distance between the parallel axes)
Momentum $\mathbf{p}$	$\mathbf{p} \equiv \sum_i m_i \mathbf{v}_i = M\mathbf{v}_{\text{cm}}$
Angular Momentum $\mathbf{L}$	$\mathbf{L} \equiv \sum_i \mathbf{R}_i \times m_i \mathbf{v}_i$ ( $= I\boldsymbol{\omega}$ for a rigid body rotating around the axis used when computing $I$ )
Kinetic Energy $K$ [ $\text{J} \equiv \text{Nm}$ ]:	Moving axis: $K_{\text{tot}} \equiv \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega_{\text{cm}}^2$
	Fixed axis A: $K_{\text{rot,A}} \equiv \frac{1}{2} I_A \omega_A^2$
<b>Forces</b> [ $\text{N} \equiv \text{kgm/s}^2$ ] <b>Torques</b> [ $\text{Nm}$ or $\text{J}$ ] and consequences:	$\sum_i \mathbf{F}_i = m\mathbf{a} = d\mathbf{p}/dt$ ; $\mathbf{F}_{A \text{ on } B} = -\mathbf{F}_{B \text{ on } A}$ ; $\sum_i \boldsymbol{\tau}_i = d\mathbf{L}/dt = I\boldsymbol{\alpha}$  In PHY131, rotations will always be in a fixed plane. The vectors $\boldsymbol{\tau}$ , $\boldsymbol{\omega}$ , etc. will therefore have a fixed direction perpendicular to the plane, and you do not need to worry about direction, just <b>sign</b> . Conventionally, counterclockwise is positive, but you may <b>specify</b> your own convention.

Force of <b>Gravity</b> between $M$ and $m$ , at center-to-center distance $r$	$\mathbf{F}_G = GMm/r^2$ (attractive! $G=6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ ); near sea level: $\mathbf{F}_G = mg(-\mathbf{j})$ (downwards; $g=9.80 \text{ m/s}^2$ ). Memorize $9.80$ , but not the numerical value $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
Force of a <b>Spring</b> (spring constant $k$ ):	$\mathbf{F}_s = -k\mathbf{x}$ (opposes compression/stretch $\mathbf{x}$ )
Torque:	$\boldsymbol{\tau} \equiv \mathbf{R} \times \mathbf{F}$ ( $\tau = RF \sin \theta_{\mathbf{R}, \mathbf{F}}$ ; direction: right-hand rule)
Equilibrium & Collisions:	$\sum_i \mathbf{F}_i = d\mathbf{p}_{\text{tot}}/dt = 0$ and $\sum_i \boldsymbol{\tau}_i = d\mathbf{L}_{\text{tot}}/dt = 0 \Rightarrow \Delta \mathbf{p}_{\text{tot}} = \Delta \mathbf{L}_{\text{tot}} = 0$ <i>Elastic</i> : $K$ is conserved; <i>Completely Inelastic</i> : objects stick together afterwards
Work done by a torque $\boldsymbol{\tau}_F$ over a rotation by angle $\theta$ :	$W_F \equiv \int \boldsymbol{\tau}_F \cdot d\boldsymbol{\theta}$
Power $P$ [ $W \equiv J/s$ ]:	$P_F \equiv dW_F/dt = \mathbf{F} \cdot \mathbf{v}$ or $\boldsymbol{\tau}_F \cdot \boldsymbol{\omega}$
Potential Energy $U$ of <b>conservative</b> forces $\mathbf{F}$	$U_F = -W_F$ ; e.g. $U_G = -GMm/r$ , $U_G$ (near earth) $= mgy$ ; $U_s = \frac{1}{2}kx^2$
Harmonic oscillator $m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt}$ The “circle of reference” is a picture where the oscillator displacement $x$ is horizontal, while $-v/\omega$ is vertical. The oscillator is then “visualized” as a point on a circle of radius $A$ , whose angular motion $\theta(t)$ is given by $\theta = \omega t + \phi$ , and $x(t) = A \cos \theta$ .	$x(t) = A \cos(\omega t + \phi)$ where $\omega = \sqrt{k/m}$ if the damping coefficient $b=0$ . $\omega = 2\pi f = 2\pi/T$ . $A = \sqrt{x_0^2 + (v_0/\omega)^2}$ ; $\phi = -\tan^{-1}(v_0/\omega x_0)$ when $b \neq 0$ the oscillations die exponentially as $e^{-\gamma t}$ where you do <b>not</b> need to memorize formulas like $\gamma = b/2m$ or the formula for the shifted resonant frequency.
Pendulum $I \frac{d^2\theta}{dt^2} = -MgL_{CM} \sin \theta \approx -MgL_{CM} \theta$ The small angle approximation is used in the second formula.	$\theta(t) = \Theta \cos(\omega t + \phi)$ where $\omega = \sqrt{MgL_{CM}/I}$ for the general pendulum, which becomes $\omega = \sqrt{g/L}$ for the simple pendulum. These results depend on the angular amplitude $\Theta$ being small.