Physics 555 Problem set #3 Due Friday Sept. 23

1. **Infrared activity**. CO_2 is a linear molecule, with 4 normal modes of vibration (two are degenerate bending modes.) (a) By symmetry it is easy to deduce unambiguously the eigenvectors of the two non-degenerate modes, and to chose a pair of eigenvectors for the degenerate modes. Three perpendicular mirror planes are sufficient to distinguish the symmetries of these modes from each other. Explain. (b) Why does CO_2 (and also N_2 and O_2) violate the "3n-6 rule"? (c) Why is CO_2 (but not N_2 and O_2) a "greenhouse gas"?

2. Kittel p. 103 problem 6

6. *Atomic vibrations in a metal.* Consider point ions of mass M and charge e immersed in a uniform sea of conduction electrons. The ions are imagined to be in stable equilibrium when at regular lattice points. If one ion is displaced a small distance r from its equilibrium position, the restoring force is largely due to the electric charge within the sphere of radius r centered at the equilibrium position. Take the number density of ions (or of conduction electrons) as $3/4\pi R^3$, which defines R. (a) Show that the frequency of a single ion set into oscillation is $\omega = (e^2/MR^3)^{1/2}$. (b) Estimate the value of this frequency for sodium, roughly. (c) From (a), (b), and some common sense, estimate the order of magnitude of the velocity of sound in the metal.

3. Ibach and Lüth, problem 4.1 p. 104

4.1 Localized vibrations in a crystal can be represented by a superposition of phonon modes with different wave vectors. Show that the center of gravity of such a wave packet moves with the group velocity $v_g = d\omega/dq$.

Of course, it's the center of energy, not gravity. You should construct a localized disturbance as a superposition of propagating plane-wave normal modes centered around a central wave-vector q with central frequency ω_q.

4. Ibach and Lüth, problem 4.3 p. 104

4.3 Calculate the eigenfrequency of a mass defect $M \neq m$ in a linear chain at the position $n = 0$ by invoking the ansatz $u_n = u_0 \exp(-\kappa |n| - i\omega t)$ for the displacements. For which range of M do localized vibrations exist?

Use the simplest possible linear chain, all masses equal except the one defect.