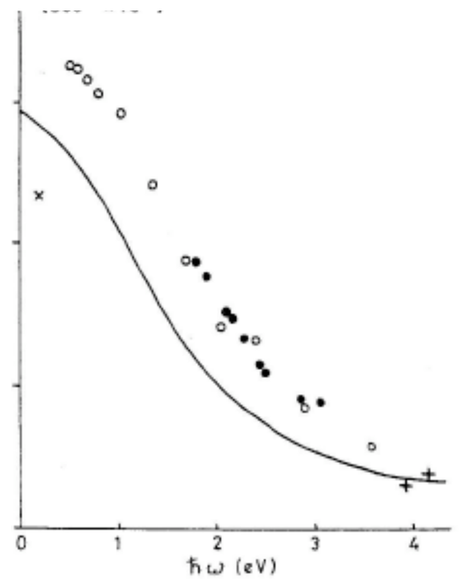
PHY 555 Problem set #8 Due Friday Nov. 18, 2011

1. The resistivity of pure metallic sodium (Na) at 300K is 4.6 μΩcm. What is the corresponding relaxation time and mean free path?

2. The figure shows the measured ac conductivity ($\sigma(\omega)$)of liquid mercury (Hg) at room temperature. The units on the vertical scale are not important. You can ignore the solid line and just consider the data points. The density is 13.6 g/cm^3. You can assume two free electrons per atom. The measured resistivity is 85 μΩcm. From this information, make a free electron model and compute the corresponding mean free path and relaxation rate $1/\tau$. Is this consistent with the figure?



3. Here is the usual model for a solid with dilute "substitutional" impurities. "Substitutional" means that the impurities replace a particular atom type and sit where the regular atom would ordinarily be. For example, NbN is an "intermetallic" compound, a metal, and an important superconductor. It might be intentionally alloyed with 2% of Zr. It would be a good first approximation that the Zr impurities sit only on the Nb sites, and otherwise have random locations. These impurities will scatter the Bloch electrons of the NbN host. Suppose the Nb atom at the origin is replaced by a Zr atom. The perturbation felt by the electrons is a local potential energy $V_1(\mathbf{r})$. The total perturbation felt by the electrons is then

$$H' = V_{\text{tot}}(\vec{r}) = \sum_i^{\text{imp}} V_1(\vec{r} - \vec{R}_i)$$

with the impurity sites $\mathbf{R}_i$ randomly distributed over the Nb sublattice.

(a) Show that the matrix element of the perturbation, $\langle \mathbf{k}'|V_{\text{tot}}|\mathbf{k}\rangle$, between host Bloch states $|\mathbf{k}\rangle$, can be written as the matrix element $\langle \mathbf{k}'|V_1|\mathbf{k}\rangle$ for an impurity at the origin, multiplied by an impurity structure factor

$$S(\vec{k} - \vec{k}') = \sum_i^{\text{imp}} e^{i(\vec{k}-\vec{k}')\cdot\vec{R}_i}$$

A sum over random phases $\exp(i\phi)$ is a 2d random walk, in the complex plane, of steps of unit length.

(b) The scattering probability per unit time for a Bloch electron $|\mathbf{k}\rangle$ to go to a different Bloch state is

$$1/\tau_k = \frac{2\pi}{\hbar} \sum_{k'} \left| \left\langle \vec{k} \middle| V_{\text{tot}} \middle| \vec{k}' \right\rangle \right|^2 \delta(\varepsilon_k - \varepsilon_{k'})$$

This involves the squared impurity structure factor. Show that this squared structure factor equals N_{imp}, the number of impurities, independent of **k,k'**. Also show that

$\langle \mathbf{k}' | V_1 | \mathbf{k} \rangle$ is proportional to 1/N, where N is the number of cells in the whole crystal, so that $n_{imp} = N_{imp}/N$ controls the scattering. The second factor of 1/N balances the sum over k'.

4. [Continuation of 3.]

(a) Explain why in a metal with T not too high, only states near the Fermi surface need to be considered in the conduction process.

(b) In a free electron model for metallic Na, suppose that there are potassium (K) impurities at the level of 2%, and that the matrix element $\langle \mathbf{k}' | V_1 | \mathbf{k} \rangle$ for scattering from one of these is independent of **k,k'** and has the value 1 eV times V_{atom}/V (the ratio of the volume per atom to the total sample volume. Compute the corresponding mean free path of an electron at the Fermi level.