Physics 556 Spring 2007

Homework assignment #1 -- Due Wednesday Jan. 31, 2007

1. The most important material is crystalline silicon, with the diamond structure. This is a cubic crystal structure. But the atoms sit in the centers of tetrahedra, and the atom sites have neither inversion symmetry nor 4-fold rotations. If there is an inversion center, where is it? If there is a 4-fold rotation axis, is it a simple or a screw rotation, and where is it?
2. Consider an ammonia molecule NH_3. Let the symmetry axis be along z, with the N atoms at $z=(3/17)u$, and the H atoms lie in the plane $z=-(14/17)u$, so that the center of mass is at the origin. The distance u is perhaps about 0.3 Angstrom. Obviously the plane $z=0$ is not a plane of mirror symmetry. But the underlying Hamiltonian does have mirror symmetry! If the z-coordinate of every nucleus is replaced by its negative, the value of H is unchanged. Therefore the "ground state" described above breaks the mirror symmetry of H. Perhaps this is not the "exact" ground state? Does the "exact" ground state respect mirror symmetry? If so, how does this happen?
3. In class we discuss the problem of 3 classical point masses ("atoms") on a line with fixed boundaries (Lax's book, Fig. 1.1.1). Consider instead the case where the 3 atoms, of mass M, are constrained to a circle, and are connected by identical springs with spring constant K. There are three translational degrees of freedom, called x_1, x_2, x_3, and 3 normal modes of oscillation in harmonic approximation. The potential energy is $V=(K/2)[(x_2-x_1)^2+(x_3-x_2)^2+(x_1-x_3)^2]$. The coordinates have origins which are separated by 1/3 of the circumference. Find the formulas for the normal mode frequencies $\omega_1, \omega_2, \omega_3$, using symmetry arguments where appropriate, and try to use symmetry to classify the normal modes. There are many ways to do this, and all are acceptable. Later, in class, an method will be discussed which uses the discrete translational symmetry (an alternative interpretation of the rotational symmetry evident here.)