PHY 556 Spring 2007 Homework #5 due Monday Mar. 26, 2007

1. Lax p. 105. By "independent" Lax means "linearly independent."

3.1.1. Given a group G of g elements $A_1, A_2, \ldots, A_g$ and a sufficiently *arbitrary* basis function Ψ such that the set of g basis functions $\Psi_i = A_i\Psi$ is an independent set, show that the Ψ_i form a basis for a representation

$$R\Psi_j = \sum \Psi_i D_{ij}^{\text{reg}}(R)$$

known as the *regular representation*, whose elements are 1 or 0 with

$$D_{ij}^{\text{reg}}(R) = \delta(R, A_j A_i^{-1})$$

2. Lax p. 106.

3.1.2. (*a*) Show that for the regular representation

$$\chi^{\text{reg}}(E) = g, \qquad \chi^{\text{reg}}(R) = 0 \quad \text{for} \quad R \neq E.$$

(*b*) Show that representation Γ_i of dimension l_i is contained l_i times in the regular representation.

(*c*) Prove Burnside's theorem, $\Sigma(l_i)^2 = g$ (Eq. 1.5.9).

3. Lax p. 131

4.3.2. Show that a measurement of electrical conductivity in a crystal of cubic symmetry is independent of the orientation of the crystal. Is this also true of optical absorption?

4. Lax p. 131

4.3.3. Show that the presence of a reflection plane containing the principal axis forces c to equal 0 in Eq. 4.3.10.

Eq. 4.3.10 applies correctly to the group C_3, not C_{3v}, as this problem clarifies.

$$\begin{bmatrix} a & c & 0 \\ -c & a & 0 \\ 0 & 0 & b \end{bmatrix} \qquad (4.3.10)$$