8/30/01. **Due:** Thursday, September 13, 2001 (in class).

Physics 503: Methods of Mathematical Physics

Homework 1

Exercise 1

Calculate real and imaginary parts of the following complex numbers:

$a)$ $2+\sqrt{17}i$ $\quad$ $b)$ $(\sqrt{2}-\sqrt{3}i)^2$ $\quad$ $c)$ $\frac{2+3i}{5-i}$

$d)$ $\left(\frac{1+i}{5}\right)^{17}$ $\quad$ $e)$ $(2+5i)^{30}$ $\quad$ $f)$ $\left(\frac{1+i}{5}\right)^{17}+(2+5i)^{30}$

Exercise 2

Find $\cos(3\theta)$ in terms of $\cos\theta$ using de Moivre's formula and identity $\sin^2\theta = 1-\cos^2\theta$.

Exercise 3

Consider the sequence defined by recurrent relation and initial conditions.

$$\begin{aligned} F_k &= 2F_{k-1} - 2F_{k-2}, \\ F_0 &= 1, \\ F_1 &= 5. \end{aligned}$$

Write down the geometric sequence ansatz and find the roots of the corresponding quadratic equation. Write down the formula for F_k using initial conditions. Analyze the result using de Moivre's formula. What is the value (order of magnitude) of F_{103}?

Exercise 4

Find the value of the following sum in a compact form

$$1 + r\cos\theta + r^2\cos 2\theta + \ldots + r^n\cos n\theta.$$

Hint: Write it as a real part of a complex geometric sequence using de Moivre's formula, sum it up, and find the real part of the result.

Exercise 5

Calculate real and imaginary parts of the principal value of the following complex numbers:

$a)$ $\ln(1+\sqrt{3}i)$ $\quad$ $b)$ $\ln(-5)$

$c)$ 2^{-i} $\quad$ $d)$ $(1-3i)^{1/3}$

Exercise 6

Find "all" multiple values (in arbitrary form) of the following expressions

$a)$ $\ln(1+\sqrt{3}i)$	$b)$ $1^{3/5}$	$c)$ $(1+\sqrt{3}i)^{1/3}$
$d)$ $\left(z^{1/2}\right)^{1/3}$	$e)$ $\left(z^{5/2}\right)^{2/5}$	$f)$ $\ln(\ln i)$