10/18/01. **Due:** Thursday, November 1, 2001 (in class).

Physics 503: Methods of Mathematical Physics

Homework 5

All these problems are two-dimensional. Charge means 2d charge etc.

Exercise 1

Find the equipotentials, streamlines, and vector intensity ($\vec{\nabla}\phi$) for the field whose complex potential $\Omega = \frac{1}{z^2}$.

Exercise 2

The equipotential lines of a fluid are the circles $x^2 + y^2 = 2ax$. Find the ratio of the magnitudes of the intensity of the field at the points $(2a, 0)$ and (a, a).

Exercise 3: Method of images

Find the force acting on electric charge q (force is equal $-q\vec{\nabla}\phi$) inside the right angle corner surrounded by metal (see figure).

Exercise 4: Streamlining the parabola

Find the velocity field for the flow streamlining the parabolic surface given by $y^2 = 2px$ with the given velocity at infinity $v(x \to -\infty) = v_\infty$ (see figure).

Hint: use the mapping $\zeta = \sqrt{z - \frac{p}{2}}$.

Exercise 5 (CKP, page 159, problem 1)

a) Show that Joukowsky transformation $z = \frac{1}{2}a\left(\zeta + \frac{1}{\zeta}\right)$ can be expressed in the useful alternative form $\frac{z-a}{z+a} = \left(\frac{\zeta-1}{\zeta+1}\right)^2$.

b) Show that the transformation doubles angles at the points $\zeta = \pm 1$.

Exercise 6

Using Joukowsky transformation find how a uniform electric field $\vec{E} = -\vec{\nabla}\phi = const$ is perturbed by metal circle of unit radius.

Exercise 7

Find the complex potential for the plane flow of a fluid flowing from the left half-plane into the right half-plane through an aperture in the imaginary axis between the points $-i$ and i (see figure). The net flow across the aperture is Q $(=\int_{-1}^{1}\phi_x dy)$.

Hint: Consider $\zeta = \ln(z+\sqrt{z^2+1})$ with properly chosen branching lines.

Exercise 8 (Not for credit)

Find the electric field (complex intensity $\Omega'(z)$) produced by a metal square of size a (see figure) and charge q at distanses $|z| \gg a$. Keep the next to a leading term in the expansion in $a/|z|$.

Hint: Use the Schwartz-Christoffel mapping to the exterior of the unit circle. Expand in $a/|z|$.