11/01/01. **Due:** Thursday, November 15, 2001 (in class).

Physics 503: Methods of Mathematical Physics

Homework 6

Exercise 1

Find the nature of each singularity (including the point at infinity) of each of the following functions.

a) $\frac{e^{1/(z-1)}}{e^z-1}$, *b)* $\frac{1}{(\sin z+\cos z)^3}$, *c)* $z^2 e^{-z}$, *d)* $\frac{z^{2n}}{(1+z^n)^2}$, *e)* $\frac{\ln(z-1)}{(z+1)^2}$.

Evaluate the residues at each isolated singularity. Always include the point at ∞ in your considerations.

Exercise 2

Find the conformal mapping of the upper half-plane onto itself which maps the points $\infty, 0, 1$ onto $0, 1, \infty$ respectively.

Exercise 3

Find the complex potential and the stream lines for the plane flow of a liquid in the first quadrant when there is a source of strength Q at $z = 1 + i$ and a sink of equal strength at $z = 0$.

Exercise 4 (CKP, page 244, problem 4a)

Use integration by parts to obtain asymptotic expansions, valid for large x, for the integral

$$I(x) = \int_0^1 (\cos t + t^2) e^{ixt}\, dt.$$

Exercise 5 (CKP, page 244, problem 4b)

Use integration by parts to obtain asymptotic expansions, valid for large x, for the integral

$$I(x) = \int_0^1 \frac{e^{ixt}}{\sqrt{t}}\, dt.$$

Hint: Write $\int_0^1 = \int_0^\infty - \int_1^\infty$.

Exercise 6 (CKP, page 254, problem 5ac)

Obtain the first few terms of the asymptotic expansions, as $x \to \infty$, of

$$a) \quad I(x) = \int_0^\pi \frac{e^{-xt^2}}{t^{1/3}} \cos t \, dt, \qquad\qquad b) \quad I(x) = \int_0^1 t^x \sin^2 t \, dt.$$

Exercise 7 (BO 6.26)

a) Obtain three terms of the asymptotic expansion of $I(x) = \int_0^{\pi/2} e^{-x\tan^2\theta} \, d\theta$ as $x \to \infty$.

b) Find the leading behavior of $I(x) = \int_0^{2\pi} (1+t^2) e^{x\cos t} \, dt$ as $x \to \infty$. Note that two maxima contribute to this leading behavior.

Exercise 8 (BO 6.28abc)

Find the leading behaviors of

$$\begin{aligned} a) \quad I(x) &= \int_0^{\pi/2} \sqrt{\sin t} \, e^{-x\sin^4 t} \, dt \quad \text{as } x \to \infty; \\ b) \quad I(x) &= \int_0^1 \sqrt{t(1-t)} \, (t+a)^{-x} \, dt \quad \text{as } x \to +\infty \text{ with } a > 0; \\ c) \quad I(x) &= \int_0^{\pi/4} \sqrt{\tan t} \, e^{-xt^2} \, dt \quad \text{as } x \to +\infty. \end{aligned}$$

The problems referred to as BO are taken from a (very good) book:

Carl M. Bender and Steven A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill, Inc., New York, 1978.