12/11/01. **Not for credit .**

Physics 503: Methods of Mathematical Physics

Homework 9

Exercise 1

Find the nature of each singularity (including the point at infinity) of each of the following functions.

$$a) \quad \frac{\sqrt{z(1-z)}}{(e^z-3)^2}, \qquad b) \quad z^2 e^{\frac{1}{z}}.$$

Evaluate the residues at each isolated singularity. Always include the point at ∞ in your considerations.

Exercise 2

Evaluate the following integral

$$I = \int_0^\infty \frac{\sin ax}{x(1+x^4)}\, dx.$$

Exercise 3

Find the flow of a fluid (complex potential) in the sector $0 < \arg z < \pi/3$ produced by a source of strength Q concentrated at the point $z_0 = ae^{i\pi/6}$.

Exercise 4

Find the leading behavior of the following integral as $x \to +\infty$.

$$\int_{-1}^{1} \cos\left[x\left(1 - \frac{t^2}{2} - \cos t\right)\right](\cosh t - 1)\, dt.$$

Exercise 5

Calculate the following integral (exactly)

$$\int_0^\infty \frac{e^{-px} dx}{\sqrt{x(x+a)}}.$$

Here $a > 0$, $p > 0$.

Hint: The answer is given in terms of Macdonald's function.

Exercise 6

Nematic is a liquid crystal characterized by an order parameter which is the unit three-component vector $\vec{n} = (n_1, n_2, n_3)$, $\vec{n}^2 = 1$ with an additional condition $\vec{n} \sim -\vec{n}$. The latter means that two unit vectors which are opposite to each other describe the same state.

What types of topological defects and textures are allowed for three-dimensional nematic? What about two-dimensional one?