

Physics 503: Methods of Mathematical Physics

Read: CKP chapter 2, sections 2-1 — 2-5.

“CKP” refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 2

Exercise 1 (CKP, page 29, problem 2)

Verify the Cauchy-Riemann equations for $(1 - z^2)^{1/2}$. At what points this function has singularities?

*Exercise 2 (CKP, page 29, problem 2)

Prove in an easy way that $(x^2 + y^2)^{1/4} \cos\left(\frac{1}{2} \arctan \frac{y}{x}\right)$ is harmonic.

Exercise 3 (CKP, page 30, problem 7)

If u and v are expressed in terms of polar coordinates (r, θ) , show that the Cauchy-Riemann equations can be written

$$u_r = \frac{1}{r}v_\theta, \quad \frac{1}{r}u_\theta = -v_r.$$

Exercise 4 (CKP, page 36, problem 3)

Show in an easy way that the integral of each of the following expressions around the circle $|z| = 1/2$ vanishes:

$$a) \frac{z+1}{z^2+z+1}, \quad b) e^{z^2} \ln(1+z), \quad c) \arcsin z.$$

Exercise 5 (CKP, page 40, problem 1)

Use Cauchy’s integral formula to evaluate the integral around the unit circle ($|z| = 1$) of

$$a) \frac{\sin z}{2z+i}, \quad b) \frac{\ln(z+2)}{z+2}, \quad c) \frac{z^3 + \operatorname{arcsinh}(z/2)}{z^2 + iz + i}.$$

***Exercise 6**

Find the principal value of the integral $\int_C \frac{\sin z}{z^2} dz$ where counterclockwise contour C is a square $ABDF$ with $A = 0$, $B = 2\pi$, $D = 2\pi(1 + i)$, and $F = 2\pi i$.

Exercise 7 (CKP, page 43, problem 1)

Find the maximum for $|z| \leq 1$ of functions

$$a) |z^2 + 2z + i|, \quad b) |\sin(z)|, \quad c) \left| \arcsin \frac{z}{2} \right|.$$