Physics 503: Methods of Mathematical Physics

Read: **CKP** chapter 2, sections 2-6 – 2-8.
chapter 3, sections 3-1, 3-2.

"**CKP**" refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 3

*Exercise 1

Show that the Cauchy-Riemann equations for modulus and argument of function $f(z) = |f|e^{i\theta}$ can be written in the form

$$(\ln|f|)_x = \theta_y, \qquad (\ln|f|)_y = -\theta_x.$$

Exercise 2 (CKP, page 56, problem 3)

Expand in powers of z the function $\sin(z + 1/z)$ in whatever annual region is closest to the origin. Express the coefficients as simple (purely real) trigonometric integrals.

Exercise 3

Discuss the character of the singularities of functions

$$a)\ \frac{1}{(z^2+2)^2(z-i)}, \qquad b)\ \cot^2(z), \qquad c)\ \frac{1}{\sin z^2}, \qquad d)\ \frac{1}{(z^2-1)^{1/2}+z+i}.$$

Include the point at ∞ in your considerations.

Exercise 4

Discuss the character of the singularities of the following functions ($a > 0$)

$$a)\ \frac{1}{z^2+a^2}, \qquad b)\ \frac{z^2}{z^2+a^2}, \qquad c)\ \frac{\sin(1/z)}{z^2+a^2}, \qquad d)\ \frac{ze^{iz}}{z^2-a^2}.$$

Always include the point at ∞ in your considerations. Evaluate the residues at isolated singularities (and at ∞ if it is possible).

*Exercise 5

Same as in Ex.4.

$$a)\ \frac{z^{-k}}{z+1},\quad 0<k<1, \qquad b)\ \frac{z-3}{z\sqrt{z^2-a^2}}, \qquad c)\ \frac{\ln z}{\sqrt{z^2+a^2}},$$
$$d)\ \frac{\cos az}{(z^2+1)^2}, \qquad e)\ \tan z.$$

Exercise 6

Evaluate

$$I = \int_0^\infty \frac{dx}{1+x^{2001}}.$$

Exercise 7

Evaluate

$$I = \int_0^\infty \frac{dx}{x^3+x^2+x+1}.$$

Use "logarithm trick".

Exercise 8 (CKP, page 82, problem 1)

Evaluate (using Jordan's lemma where necessary)

$$I = \int_0^\infty \frac{x\sin x}{a^2+x^2}\,dx.$$

Exercise 9 (CKP, page 89, problem 8)

Evaluate

$$I = \int_0^{2\pi} \ln(a+b\cos\theta)\,d\theta$$

for $a>b>0$.

Exercise 10 (CKP, page 90, problem 12)

Show that

$$(a)\qquad \int_0^{2\pi} e^{\cos\theta}\cos(n\theta-\sin\theta)\,d\theta = \frac{2\pi}{n!}$$
$$(b)\qquad \int_0^{2\pi} e^{\cos\theta}\sin(n\theta-\sin\theta)\,d\theta = 0$$

Exercise 11 (CKP, page 90, problem 14)

Evaluate

$$I = \int_0^{2\pi} \frac{x \sin x}{1 - 2\alpha \cos x + \alpha^2}\, dx$$

for α real and for each of the two cases $|\alpha| < 1$, $|\alpha| > 1$.

Exercise 12 (CKP, page 90, problem 16)

Evaluate

$$\begin{aligned} I &= \int_0^{\infty} \frac{\ln(1+x^2)}{1+x^2}\, dx \\ I &= \int_0^{\infty} \frac{(\ln x)^2}{1+x^2}\, dx \end{aligned}$$

*Exercise 13

Show that

$$a) \qquad \cot z = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - \pi^2 n^2}$$

$$b) \qquad \frac{1}{\sin^2 z} = \sum_{n=-\infty}^{+\infty} \frac{1}{(z - \pi n)^2}$$