# Physics 503: Methods of Mathematical Physics

**Read:** **CKP** sections 6-1 – 6-3.
**BO** sections 3.8, 6.1 – 6.4.

"**CKP**" refers to Carrier, Krook, and Pearson book.
"**BO**" refers to Bender and Orszag book.
Problems with stars are not for credit and will NOT be graded.

# Homework 5

## Exercise 1 (FS 81.1cg)

Evaluate the following integrals

$$I = \int_{-\infty}^{\infty} \frac{\cos ax}{1+x^4}\, dx,$$
$$I = \int_{-1}^{1} \frac{dx}{[(1-x)(1+x)^2]^{1/3}}.$$

## *Exercise 2

Find the conformal mapping of the upper half-plane onto itself which maps the points $\infty, 0, 1$ onto $0, 1, \infty$ respectively.

## Exercise 3

Find the complex potential and the stream lines for the plane flow of a liquid in the first quadrant when there is a source of strength $Q$ at $z = 1 + i$ and a sink of equal strength at $z = 0$.

## *Exercise 4 (CKP, page 244, problem 4a)

Use integration by parts to obtain asymptotic expansions, valid for large x, for the integral

$$I(x) = \int_0^1 (\cos t + t^2) e^{ixt}\, dt.$$

## Exercise 5 (CKP, page 244, problem 4b)

Use integration by parts to obtain asymptotic expansions, valid for large x, for the integral

$$I(x) = \int_0^1 \frac{e^{ixt}}{\sqrt{t}}\, dt.$$

*Hint*: Write $\int_0^1 = \int_0^\infty - \int_1^\infty$.

## Exercise 6 (CKP, page 254, problem 5ac)

Obtain the first few terms of the asymptotic expansions, as $x \to \infty$, of

$$a) \quad I(x) = \int_0^\pi \frac{e^{-xt^2}}{t^{1/3}} \cos t\, dt, \qquad\qquad b) \quad I(x) = \int_0^1 t^x \sin^2 t\, dt.$$

The problems referred to as FS are taken from the book:

B. A. Fuchs and B. V. Shabat, *Functions of a complex variable and some of their applications*, v. I, Pergamon press, 1964.