Physics 503: Methods of Mathematical Physics

Read: CKP chapter 2, sections 2-1 — 2-5.

“CKP” refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 2

Exercise 1 (CKP, page 29, problem 2)
Verify the Cauchy-Riemann equations for \((1 - z^2)^{1/2}\). At what points this function has singularities?

Exercise 2 (CKP, page 29, problem 2)
Prove in an easy way that \((x^2 + y^2)^{1/4} \cos \left( \frac{1}{2} \arctan \frac{y}{x} \right)\) is harmonic.

*Exercise 3 (CKP, page 30, problem 7)
If \(u\) and \(v\) are expressed in terms of polar coordinates \((r, \theta)\), show that the Cauchy-Riemann equations can be written
\[
\frac{u_r}{r} = \frac{v_\theta}{r}, \quad \frac{1}{r} u_\theta = -v_r.
\]

Exercise 4 (CKP, page 36, problem 3)
Show in an easy way that the integral of each of the following expressions around the circle \(|z| = 1/2\) vanishes:
\[ a) \quad \frac{z^2 + 1}{z^2 + z + 1}, \quad b) \quad e^{z^2} \ln(1 + z), \quad c) \quad \arcsin z. \]

Exercise 5 (CKP, page 40, problem 1)
Use Cauchy’s integral formula to evaluate the integral around the unit circle (\(|z| = 1\)) of
\[ a) \quad \frac{\sin z}{2z + 1}, \quad b) \quad \frac{\ln(z + 2)}{z + 2}, \quad c) \quad \frac{z^3 + \arcsinh(z/2)}{z^2 + z + 3}. \]
Exercise 6

Find the principal value of the integral $\int_C \frac{\sin z}{z^2} \, dz$ where counterclockwise contour $C$ is a square $ABDF$ with $A = 0$, $B = 2\pi$, $D = 2\pi(1 + i)$, and $F = 2\pi i$.

*Exercise 7 (CKP, page 43, problem 1)

Find the maximum for $|z| \leq 1$ of functions

\begin{align*}
  a) \quad |z^2 + 2z + i|, \quad b) \quad |\sin(z)|, \quad c) \quad |\arcsin \frac{z}{2}|.
\end{align*}