

Physics 503: Methods of Mathematical Physics

Read: CKP chapter 2, sections 2-6 – 2-8.
chapter 3, sections 3-1, 3-2.

“**CKP**” refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 3

*Exercise 1

Show that the Cauchy-Riemann equations for modulus and argument of function $f(z) = |f|e^{i\theta}$ can be written in the form

$$(\ln |f|)_x = \theta_y, \quad (\ln |f|)_y = -\theta_x.$$

Exercise 2 (CKP, page 56, problem 3)

Expand in powers of z the function $\sin(z + 1/z)$ in whatever annular region is closest to the origin. Express the coefficients as simple (purely real) trigonometric integrals.

Exercise 3

Discuss the character of the singularities of functions

$$a) \frac{1}{(z^2+2)^2(z-i)}, \quad b) \cot^2(z), \quad c) \frac{1}{\sin z^2}, \quad d) \frac{1}{(z^2-1)^{1/2}+z+i}.$$

Include the point at ∞ in your considerations.

*Exercise 4

Discuss the character of the singularities of the following functions ($a > 0$)

$$a) \frac{1}{z^2+a^2}, \quad b) \frac{z^2}{z^2+a^2}, \quad c) \frac{\sin(1/z)}{z^2+a^2}, \quad d) \frac{ze^{iz}}{z^2-a^2}.$$

Always include the point at ∞ in your considerations. Evaluate the residues at isolated singularities (and at ∞ if it is possible).

Exercise 5

Same as in Ex.4.

$$\begin{array}{lll} a) \frac{z^{-k}}{z+1}, & 0 < k < 1, & b) \frac{z-3}{z\sqrt{z^2-a^2}}, \\ d) \frac{\cos az}{(z^2+1)^2}, & & e) \tan z. \end{array} \quad c) \frac{\ln z}{\sqrt{z^2+a^2}},$$

Exercise 6

Evaluate

$$I = \int_0^\infty \frac{dx}{1+x^{2003}}.$$

*Exercise 7

Evaluate

$$I = \int_0^\infty \frac{dx}{x^3 + x^2 + x + 1}.$$

Use “logarithm trick”.

Exercise 8 (CKP, page 82, problem 1)

Evaluate (using Jordan’s lemma where necessary)

$$I = \int_0^\infty \frac{x \sin x}{a^2 + x^2} dx.$$

Exercise 9 (CKP, page 89, problem 8)

Evaluate

$$I = \int_0^{2\pi} \ln(a + b \cos \theta) d\theta$$

for $a > b > 0$.

*Exercise 10 (CKP, page 90, problem 12)

Show that

$$\begin{array}{l} (a) \int_0^{2\pi} e^{\cos \theta} \cos(n\theta - \sin \theta) d\theta = \frac{2\pi}{n!} \\ (b) \int_0^{2\pi} e^{\cos \theta} \sin(n\theta - \sin \theta) d\theta = 0 \end{array}$$

Exercise 11 (CKP, page 90, problem 14)

Evaluate

$$I = \int_0^{2\pi} \frac{x \sin x}{1 - 2\alpha \cos x + \alpha^2} dx$$

for α real and for each of the two cases $|\alpha| < 1$, $|\alpha| > 1$.

Exercise 12 (CKP, page 90, problem 16)

Evaluate

$$a) \quad I = \int_0^{\infty} \frac{\ln(1+x^2)}{1+x^2} dx$$

$$* \quad b) \quad I = \int_0^{\infty} \frac{(\ln x)^2}{1+x^2} dx$$

*Exercise 13

Show that

$$a) \quad \cot z = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - \pi^2 n^2}$$

$$b) \quad \frac{1}{\sin^2 z} = \sum_{n=-\infty}^{+\infty} \frac{1}{(z - \pi n)^2}$$