

# Physics 301/571: Electromagnetic Theory I

**Read: Griffiths** chapters 7.1-7.2

“G” refers to Griffiths’ book.

Problems with stars are not for credit and will NOT be graded.

## Homework 11

### Exercise 1 (G 7.5)

A battery of emf  $\mathcal{E}$  and internal resistance  $r$  is hooked up to a variable “load” resistance,  $R$ . If you want to deliver the maximum possible power to the load, what resistance  $R$  should you choose? (You can’t change  $\mathcal{E}$  and  $r$ , of course.)

### Exercise 2 (G 7.2 partial)

A capacitor  $C$  has been charged up to potential  $V_0$ ; at time  $t = 0$  it is connected to a resistor  $R$ , and begins to discharge.

(a) Determine the charge on the capacitor as a function of time,  $Q(t)$ . What is the current through the resistor,  $I(t)$ ?

(b) What was the original energy stored in the capacitor? By integrating Joule’s heating law confirm that the heat delivered to the resistor is equal to the energy lost by the capacitor.

### \*Exercise 3 (G 7.10)

A square loop (side  $a$ ) is mounted on a vertical shaft and rotated at angular velocity  $\omega$ . A uniform magnetic field  $\vec{B}$  points to the right. Find the  $\mathcal{E}(t)$  for this *alternating current* generator.

### Exercise 4 (G 7.13)

A square loop of wire, with sides of length  $a$ , lies in the first quadrant of the  $xy$  plane, with one corner at the origin. In this region there is a nonuniform time-dependent magnetic field  $\vec{B}(y, t) = ky^3t^2\hat{z}$  (where  $k$  is a constant). Find the emf induced in the loop.

### Exercise 5 (G 7.15)

A long solenoid with radius  $a$  and  $n$  turns per unit length carries a time-dependent current  $I(t)$  in the  $\hat{\phi}$  direction. Find the electric field (magnitude and direction) at a distance  $s$  from the axis (both inside and outside the solenoid), in the quasistatic approximation.

### Exercise 6 (G 7.21)

A square loop of wire, of side  $a$ , lies midway between two long wires,  $3a$  apart, and in the same plane. (Actually, the long wires are sides of a large rectangular loop, but the short ends are so far away that they can be neglected.) A clockwise current  $I$  in the square loop is gradually increasing:  $dI/dt = k$  (a constant). Find the emf induced in the big loop. Which way will the induced current flow?

### \*Exercise 7 (G 7.23 modified)

Find the self inductance of the long rectangular loop of the width  $d$  and length  $l \gg s$ . Assume that the wire has a tiny radius  $\epsilon$ .

### \*Exercise 8

A circuit consists of a capacitor  $C$ , an inductor  $L$ , a battery with emf  $\mathcal{E}$ , and an open switch connected in series. At time  $t = 0$  the switch is closed.

- (a) Find the current  $I(t)$  in the circuit as a function of time.
- (b) Find the voltage  $V_C(t)$  across the capacitor as a function of time.
- (c) Find the voltage  $V_L(t)$  across the inductor as a function of time.

### Exercise 9 (G 7.28)

A long cable carries current in one direction uniformly distributed over its (circular) cross section. The current returns along the surface (there is a very thin insulating sheath separating the currents). Find the self-inductance per unit length.

*Hint:* calculate the energy of magnetic field.