Physics 301/571: Electromagnetic Theory I

Read: Griffiths chapter 2.2-2.3

“G” refers to Griffiths’ book.
Problems with stars are not for credit and will NOT be graded.

Homework 4

Exercise 1 (G 2.12 partial)
Use Gauss’s law to find the electric field inside and outside of a uniformly charged solid sphere whose radius is $R$ and whose total charge is $q$.

Exercise 2 (G 2.21)
Find the potential inside and outside of a uniformly charged solid sphere whose radius is $R$ and whose total charge is $q$. Use infinity as your reference point. Compute the gradient of $V$ in each region and compare the result with the result of Ex. 1. Sketch $V(r)$.

Important: Solve this problem using the integral formula for a potential of a distributed charge.

Exercise 3 (G 2.13)
Find the electric field a distance $s$ from an infinitely long straight wire, which carries a uniform line charge $\lambda$.

Exercise 4 (G 2.22)
Find the potential a distance $s$ from an infinitely long straight wire that carries a uniform line charge $\lambda$. Compute the gradient of your potential, and compare the result with the result of Ex. 3.

Important: Solve this problem using the integral formula for a potential of a distributed charge.
Exercise 5

Sketch field lines of three electric charges \( q, q, \) and \(-3q\) placed in the vertices of an equilateral triangle. Please, show only field lines lying in the plane of triangle (two-dimensional picture).

*Exercise 6

Consider two charges \( q \) and \(-q\) placed in the points \( \vec{l}/2 \) and \(-\vec{l}/2\) respectively.

a) Write down the potential \( V(\vec{r}) \) created by those charges at point \( \vec{r} \).

b) Expand the result of a) in \( a/r \) and find the potential \( V(\vec{r}) \) in the limit \( a \ll r \) leaving only the first non-vanishing term of the expansion.

c) Introduce the “electric dipole moment” \( \vec{p} \equiv q\vec{l} \) and express the result of b) in terms of \( \vec{p} \). You have obtained the potential of an electric dipole.

d) Compute the gradient of the potential from c) and find the corresponding electric field. You have obtained the electric field of an electric dipole.

*Hint:* Use the following approximate formula

\[
|\vec{r} - \vec{a}|^\alpha = (r^2 - 2\vec{r} \cdot \vec{a} + a^2)^{\alpha/2} = r^{\alpha} \left( 1 - 2\frac{\vec{r} \cdot \vec{a}}{r^2} + \frac{a^2}{r^2} \right)^{\alpha/2} \approx r^{\alpha} \left( 1 - \alpha \frac{\vec{r} \cdot \vec{a}}{r^2} \right).
\]