

## Physics 501: Classical Mechanics

Read: LL 27-30; JS 7; G 11; LL7 I,II.

### Homework 10

#### Exercise 1

Find the 2-cycle for the logistic equation  $x_{n+1} = ax_n(1 - x_n)$ . Analyze the stability of this cycle (find the range of  $a$  where it is stable).

#### Exercise 2

The standard map (corresponding to a kicked rotator) is given by

$$\begin{aligned}\phi_{n+1} &= (\phi_n + J_{n+1}) \bmod 2\pi, \\ J_{n+1} &= \epsilon \sin \phi_n + J_n.\end{aligned}$$

Here  $\phi$  and  $J$  are the angle and the velocity of the rotator, and  $\epsilon$  is the strength of kicks (control parameter). Using Maple, Mathematica, or similar software draw Poincare maps (trajectories in  $\phi - J$  plane) for this system for several values of  $\epsilon$ , e.g.,  $\epsilon = 0, 0.8, 5$ .

#### Exercise 3

Consider a periodically driven pendulum described by

$$\ddot{\phi} + \omega_0^2 [1 + h \cos(2\omega_0 + \epsilon)t] \sin \phi = 0.$$

Here  $h \ll 1$ .

a) Consider a non-driven pendulum ( $h = 0$ ). Due to the nonlinearity of the problem the frequency of oscillations is amplitude dependent. Find the correction to the frequency of oscillations for the finite (but small) amplitude of oscillations  $\phi_0 \ll 1$ .

b) Assume now that  $h \ll 1$  but is not zero. At small  $\epsilon$  the parametric resonance occurs. State the condition (on  $\epsilon$ ) for a parametric resonance.

c) Let us now assume that  $\epsilon = 0$ . Parametric resonance occurs and amplitude of oscillations grows with time. Due to the nonlinearity of the problem the basic frequency of oscillations changes with the amplitude of oscillations and at some point the parametric resonance condition will be violated. Estimate the maximal (saturation) amplitude  $\phi_{max}$  of oscillations as a function of  $h \ll 1$ .

### Exercise 4

Determine the deformation of a long rod (with length  $L$ ) rotating with frequency  $\Omega$  around its end (the axis of rotation is orthogonal to the rod).

### Exercise 5

Calculate the deformation law  $h(x)$  for a thin, heavy, elastic board which is weakly bent by the Earth gravity (see Figure 1). Assume that the Young's modulus  $E$ , the mass of the board, and its dimensions are known. How does the maximal deflection of the board scale with  $L$ ?

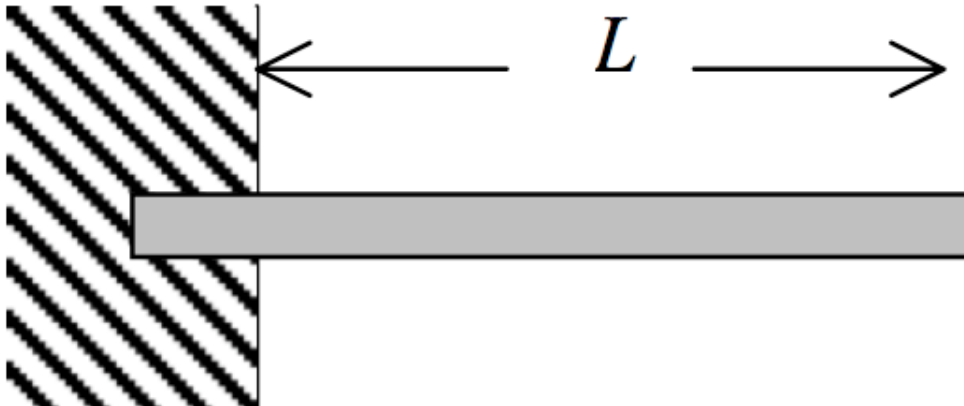


Figure 1: To Exercise 5.

### Exercise 6 (KdV equation)

Consider the continuous system which is defined by a Hamiltonian (given as a functional of the field  $u(x)$ )

$$H[u] = \int dx \frac{1}{6} [u^3 + 3u_x^2]$$

and by Poisson bracket

$$\{u(x), u(y)\} = \partial_x \delta(x - y).$$

a) Derive the equation of motion for the field  $u(x, t)$ .

b) Assume that the system has periodic boundary conditions. Writing  $u(x) = \sum_p u_p e^{ipx}$  find the Poisson bracket  $\{u_p, u_q\}$ . Suggest the canonically conjugated coordinates and momenta for the dynamical system.

### Exercise 7

Find the stationary shape of the surface of an incompressible fluid rotated around the vertical axis with a constant angular velocity  $\Omega$ .