

Physics 501: Classical Mechanics

Read: LL 20-25;

Problems with stars are not for credit and will NOT be graded.

Homework 5

Exercise 1

Beam and target particles interact through the repulsive potential $U(r) = -\frac{k}{r^2}$. Find the differential cross section at small angles θ .

Exercise 2

Two blocks of masses m_1 and m_2 are connected by the spring which has potential energy $b(s^2 - a^2)^2$ and are free to move along the horizontal frictionless table. Here a, b are positive constants and s is the distance between blocks.

a) Write down the Lagrangian of this system in terms of \vec{R}_1 , and \vec{R}_2 - the two-dimensional position-vectors of blocks.

b) Assume that the equilibrium positions of blocks are $(\pm a/2, 0)$ and introduce the deviations $\vec{r}_{1,2}$ from equilibrium positions. Write down the harmonic approximation of the Lagrangian of a) in terms of these deviations.

c) Find all eigenfrequencies of the system.

Exercise 3

For the system of Ex. 2.

a) Find all normal modes of the system and express the Lagrangian of b) in terms of normal coordinates $Q_{1,2,3,4}$.

b) Explain the peculiarities of the eigenspectrum found in Ex. 2 using the results of a) and symmetry considerations.

Exercise 4

N identical particles of mass m are free to move along the circle of radius R without any friction. Particles are connected by identical springs of spring constants k that lie along the arcs of the circle.

- Write down the Lagrangian of the system using the angular deviations x_n ($n = 1, \dots, N$) of particle from the equilibrium N -gonal configuration.
- Find all eigenfrequencies of the system if $N = 2$.
- Find all eigenfrequencies of the system if $N = 3$.
- Explain the presence of zero modes in b), c).

Exercise 5

For the system of Ex. 4.

- Find all eigenfrequencies of the system for general N .
- Find normal coordinates and express the Lagrangian of the system in terms of normal coordinates for general N .

Hint: It is convenient to use $\tilde{x}_p = \frac{1}{\sqrt{N}} \sum_{n=1}^N x_n e^{-i\frac{2\pi}{N}np}$ with $p = 0, 1, \dots, N-1$ as coordinates instead of x_n .

- Plot the eigenfrequency $\omega(q)$ versus $q = 2\pi p/N$. What happens in the limit $N \rightarrow \infty$?

Exercise 6 (JS 4.11)

a) For the undamped, one-freedom, linear oscillator with natural frequency ω_0 , driven at frequency $\Omega \neq \omega_0$, find the rate at which energy is pumped into the oscillator. Find the maximum energy of an oscillator starting from rest at $q = 0$.

- Do the same when the driving frequency is ω_0 .