10/26/2005. **Due:** Wednesday, Nov. 2, 2005 (in class).

# Physics 501: Classical Mechanics

*Read:* **LL** 31-39;

Problems with stars are not for credit and will NOT be graded.

# Homework 7

## Exercise 1 (G 5.6)

A uniform right circular cone of height $h$, half-angle $\alpha$, and density $\rho$ rolls on its side without slipping on a uniform horizontal plane in such a manner that it returns to its original position in a time $\tau$. Find expressions for the kinetic energy and the components of the angular momentum of the cone.

## Exercise 2 (G 5.13)

A uniform bar of mass $M$ and length $2l$ is suspended from one end by a spring of force constant $k$. The bar can swing freely only in one vertical plane, and the spring is constrained to move only in the vertical direction. Set up the equations of motion in the Lagrangian formulation.

## Exercise 3

Derive the Lagrangian of a rigid body with a fixed point in terms of the SO(3) matrix $g$ ($g^T = g^{-1}$, $\det g = 1$) which gives the orientation of the body with respect to a laboratory frame. Namely,

a) Show that the rotational kinetic energy (in this case it is the Lagrangian) is given by

$$L = \frac{1}{4}\mathrm{tr}\,\left(\hat{I}\right)\mathrm{tr}\,\left(\dot{g}^T\dot{g}\right) - \frac{1}{2}\mathrm{tr}\,\left(\hat{I}\dot{g}^T\dot{g}\right), \tag{1}$$

where dot means time derivative and $\hat{I}$ is the inertia tensor.

b) Show that the conventional angular velocity is given by $\Omega_k = -\frac{1}{2}\epsilon^{ijk}(g^T\dot{g})_{ij}$, where $\epsilon^{ijk}$ is a Levi-Civita tensor and $(\ldots)_{ij}$ means the matrix element of $(\ldots)$.

c) Express the result of a) in terms of the angular velocity using the result of b) and compare with the standard expression.

## *Exercise 4

a) The Lagrangian (1) has a continuous symmetry $g \to hg$, where $h$ is an arbitrary (constant) SO(3) matrix. Using the Noether's theorem find the corresponding conserved quantity. How is this quantity called? What is the nature of this symmetry?

b) If $\hat{I}$ is the inertia tensor of a symmetric top there is an additional symmetry in (1). What is the corresponding conserved quantity?

## Exercise 5

You are trying to hit some hard obstacle with a uniform stick of the length $L$ holding it at the end. What should be the distance from the end of the stick to the "hitting point" so that you feel the most comfortable (to have the least recoil to the hand)?

## Exercise 6 (G 4.22)

A particle is thrown up vertically with initial speed $v_0$, reaches a maximum height and falls back to ground. Show that the Coriolis deflection when it again reaches the ground is opposite in direction, and four times greater in magnitude, than the Coriolis deflection when it is dropped at rest from the same maximum height.