Physics 501: Classical Mechanics

Read: **LL** 40-45;

Homework 8

Exercise 1

A Hamiltonian of a charged particle moving in a constant uniform magnetic field $\vec{B}$ is given by

$$H = \frac{(\vec{p} - \frac{e}{c}\vec{A})^2}{2m},$$

where $\vec{A}(\vec{r})$ is a vector potential corresponding to the constant magnetic field ($\vec{\nabla} \times \vec{A} = \vec{B}$).

a) Evaluate the Poisson brackets $[v_i, v_j]$ of the Cartesian components of the velocity of the particle $v_i = \dot{x}_i$, $i = 1, 2, 3$.

b) Rewrite the Hamiltonian in terms of particle's velocity $\vec{v}$ and find $\frac{d\vec{v}}{dt}$ commuting the Hamiltonian with $\vec{v}$ and using the results of a).

Exercise 2 (G 9.30)

A system of two degrees of freedom is described by the Hamiltonian

$$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2.$$

Show that $F_1 = \frac{p_1 - a q_1}{q_2}$ and $F_2 = q_1 q_2$ are constants of motion. Are there any other independent algebraic constants of motion? Can any be constructed from Jacobi's identity?

Exercise 3

Hamiltonian of the Kepler problem is given by

$$H = \frac{\vec{p}^2}{2m} - \frac{\alpha}{r}.$$

a) Show from the Poisson bracket condition for conserved quantities that the Laplace-Runge-Lenz vector $\vec{A} = \vec{p} \times \vec{M} + \alpha m \hat{r}$ is a constant. Here $\vec{M} = \vec{r} \times \vec{p}$ is an angular momentum and $\hat{r} = \vec{r}/r$.

b) Derive $[A_i, A_j]$ - the Poisson's bracket of Cartesian components of $\vec{A}$.

Hint: See G. Section 9-7.

Exercise 4 (G 9.4)

Show directly that the transformation $Q = \log(\frac{1}{q}\sin p)$, $P = q \cot p$ is canonical.

Exercise 5 (G 9.8)

Prove directly that the transformation

$$\begin{aligned} Q_1 &= q_1, &\qquad P_1 &= p_1 - 2p_2, \\ Q_2 &= p_2, &\qquad P_2 &= -2q_1 - q_2 \end{aligned}$$

is canonical and find a generating function.