Physics 501: Classical Mechanics

Read: **LL** 47-50; **JS** 6.3

Homework 9

Exercise 1

A charged particle is constrained to move in a plane under the influence of a central force potential (nonelectromagnetic) $V = \frac{1}{2}kr^2$, and a constant magnetic field $\vec{B}$ perpendicular to the plane, so that $\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$. Set up the Hamilton-Jacobi equation in plane polar coordinates. Separate the equation and reduce it to quadratures. Discuss the motion if the canonical momentum p_θ is zero at time $t = 0$.

Exercise 2

A particle moves in periodic motion in one dimension under the influence of a potential $V(x) = F|x|$, where F is a constant. Using action-angle variables find the period of the motion as a function of the particle's energy.

Exercise 3 (JS 6.25)

An object is bouncing vertically and perfectly elastically in an accelerating elevator. If the time dependence of the acceleration $a(t)$ is slow enough to satisfy the adiabatic assumption, find the maximum heights $h_{max}(t)$ that the object reaches on its bounces [$h_{max}(t)$ is measured relative to the floor of the elevator].

Exercise 4 (JS 6.21)

Find the action-angle variables for a free particle in two dimensions constrained to move inside a circular region of radius a (i.e., subjected to the potential $V(r) = 0$, $r \leq a$, and $V(r) = \infty$, $r > a$). The particle makes elastic collisions (the angle of incidence equals the angle of reflection) with the circular boundary.

*Exercise 5 (JS 6.27)

For the previous problem.

a) Show that if the radius a of the circle changes adiabatically, the energy goes as $1/a^2$.

b) Find the angle Φ that the particle's velocity makes with the wall (as a function of E and Js). Show that Φ is an adiabatic invariant.

c) Find the average force the particle applies to the wall.

d) Assume that the circle is filled with a gas of particles all at the same energy moving with equal probability at all angles with respect to the wall. Find how the pressure the gas exerts on the wall depends on a when a changes adiabatically. Find γ for the system (defined through $PV^\gamma = const$ in adiabatic process, V - volume of the system).

Exercise 6 (JS 6.6)

Use the results of the secular perturbation theory treatment of the quartic oscillator to obtain the first order correction for the frequency of a simple pendulum whose amplitude is Θ.