

Physics 503: Methods of Mathematical Physics

Read: CKP chapter 1

“**CKP**” refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 1

Exercise 1

Calculate real and imaginary parts of the following complex numbers:

$$\begin{array}{lll} a) & 2 + \sqrt{17}i & b) & (\sqrt{2} - \sqrt{3}i)^2 & c) & \frac{2+3i}{5-i} \\ d) & \left(\frac{1+i}{5}\right)^{17} & e) & (2 + 5i)^{30} & f) & \left(\frac{1+i}{5}\right)^{17} + (2 + 5i)^{30} \end{array}$$

Exercise 2

Find $\sin(3\theta)$ in terms of $\sin \theta$ using de Moivre's formula and identity $\sin^2 \theta = 1 - \cos^2 \theta$.

*Exercise 3

Consider the sequence defined by recurrent relation and initial conditions.

$$\begin{aligned} F_k &= 2F_{k-1} - 2F_{k-2}, \\ F_0 &= 1, \\ F_1 &= 5. \end{aligned}$$

Write down the geometric sequence ansatz and find the roots of the corresponding quadratic equation. Write down the formula for F_k using initial conditions. Analyze the result using de Moivre's formula. What is the value (order of magnitude) of F_{103} ?

*Exercise 4 (CKP, page 5, problem 6)

Find the value of the following sum in a compact form

$$1 + r \cos \theta + r^2 \cos 2\theta + \dots + r^n \cos n\theta.$$

Hint: Write it as a real part of a complex geometric sequence using de Moivre's formula, sum it up, and find the real part of the result.

Exercise 5

Calculate real and imaginary parts of the principal value of the following complex numbers:

$$\begin{array}{ll} a) & \ln(1 + \sqrt{3}i) \\ c) & 2^{-i} \end{array} \qquad \begin{array}{l} b) \quad \ln(-5) \\ d) \quad (1 - 3i)^{1/3} \end{array}$$

Exercise 6

Find “all” multiple values (in arbitrary form) of the following expressions

$$\begin{array}{lll} a) & \ln(1 + \sqrt{3}i) & b) \quad 1^{3/5} & c) \quad (1 + \sqrt{3}i)^{1/3} \\ d) & (z^{1/2})^{1/3} & e) \quad (z^{5/2})^{2/5} & f) \quad \ln(\ln i) \end{array}$$

*Exercise 7

Show that the cross-ratio is an invariant of fractional transformation, i.e., that

$$\frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_3)(w_2 - w_4)} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_3)(z_2 - z_4)}$$

when $w = \frac{az+b}{cz+d}$.

Exercise 8 (CKP, page 19, problem 1)

Use the cross-ratio to obtain a mapping which transforms the upper-half z plane into the interior of the unit circle in the w plane. Sketch the w images of various points and curves in the z plane, and vice versa. (*Hint:* Replace z_1, w_1 by z, w ; set z_2, z_3, z_4 equal to $-1, 0, 1$, etc. Or use point at infinity.)

*Exercise 9 (CKP, page 24, problem 13a)

Discuss the branch cut and Riemann-surface situation for the following function

$$g(z) = \sqrt{1 + \sqrt{z}}.$$