Physics 503: Methods of Mathematical Physics

Read: CKP chapter 2, sections 2-1 — 2-5.

“CKP” refers to Carrier, Krook, and Pearson book. Problems with stars are not for credit and will NOT be graded.

Homework 2

Exercise 1 (CKP, page 29, problem 2)

Verify the Cauchy-Riemann equations for \((1 - z^2)^{1/2}\). At what points this function has singularities?

Exercise 2 (CKP, page 29, problem 2)

Prove in an easy way that \((x^2 + y^2)^{1/4} \cos \left( \frac{1}{2} \arctan \frac{y}{x} \right)\) is harmonic.

*Exercise 3 (CKP, page 30, problem 7)

If \(u\) and \(v\) are expressed in terms of polar coordinates \((r, \theta)\), show that the Cauchy-Riemann equations can be written

\[
\frac{u_r}{r} = \frac{v_\theta}{r}, \quad \frac{1}{r} u_\theta = -v_r.
\]

Exercise 4 (CKP, page 36, problem 3)

Show in an easy way that the integral of each of the following expressions around the circle \(|z| = 1/2\) vanishes:

\(a) \ \frac{z+1}{z^2+z+1}, \quad b) \ e^{z^2} \ln(1 + z), \quad c) \ \arcsin z.\)

Exercise 5 (CKP, page 40, problem 1)

Use Cauchy’s integral formula to evaluate the integral around the unit circle (\(|z| = 1\)) of

\(a) \ \frac{\sin z}{2z+1}, \quad b) \ \frac{\ln(z+2)}{z+2}, \quad c) \ \frac{z^3+\text{arcsinh} (z/2)}{z^2+z+4}.\)
Exercise 6

Find the principal value of the integral \( \int_C \frac{\sin z}{z^2} \, dz \) where counterclockwise contour \( C \) is a square \( ABDF \) with \( A = 0, B = 2\pi, D = 2\pi(1 + i) \), and \( F = 2\pi i \).

*Exercise 7 (CKP, page 43, problem 1)

Find the maximum for \( |z| \leq 1 \) of functions

\[ a) \ |z^2 + 2z + i|, \quad b) \ |\sin(z)|, \quad c) \ |\arcsin \frac{z}{2}|. \]

*Exercise 8

Show that the Cauchy-Riemann equations for modulus and argument of function \( f(z) = |f|e^{i\theta} \) can be written in the form

\[ (\ln |f|)_x = \theta_y, \quad (\ln |f|)_y = -\theta_x. \]