9/11/2007. **Due:** Thursday, Sept. 20, 2007 (in class).

Physics 503: Methods of Mathematical Physics

Read: CKP chapter 1

"**CKP**" refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 1

Exercise 1

Calculate real and imaginary parts of the following complex numbers:

$a)\quad 2+\sqrt{17}i$	$b)\quad (\sqrt{2}-\sqrt{3}i)^2$	$c)\quad \frac{2+3i}{5-i}$
$d)\quad \left(\frac{1+i}{5}\right)^{17}$	$e)\quad (2+5i)^{30}$	$f)\quad \left(\frac{1+i}{5}\right)^{17}+(2+5i)^{30}$

Exercise 2

Find $\sin(3\theta)$ in terms of $\sin\theta$ using de Moivre's formula and identity $\sin^2\theta = 1-\cos^2\theta$.

Exercise 3

Consider the sequence defined by recurrent relation and initial conditions.

$$\begin{aligned} F_k &= 2F_{k-1} - 2F_{k-2}, \\ F_0 &= 1, \\ F_1 &= 5. \end{aligned}$$

Write down the geometric sequence ansatz and find the roots of the corresponding quadratic equation. Write down the formula for F_k using initial conditions. Analyze the result using de Moivre's formula. What is the value (order of magnitude) of F_{103}?

*Exercise 4(CKP, page 5, problem 6)

Find the value of the following sum in a compact form

$$1 + r\cos\theta + r^2\cos 2\theta + \ldots + r^n\cos n\theta.$$

Hint: Write it as a real part of a complex geometric sequence using de Moivre's formula, sum it up, and find the real part of the result.

*Exercise 5

Calculate real and imaginary parts of the principal value of the following complex numbers:

$a)\quad \ln(1+\sqrt{3}i) \qquad b)\quad \ln(-5)$
$c)\quad 2^{-i} \qquad d)\quad (1-3i)^{1/3}$

Exercise 6

Find "all" multiple values (in arbitrary form) of the following expressions

$a)\quad \ln(1+\sqrt{3}i) \qquad b)\quad 1^{3/5} \qquad c)\quad (1+\sqrt{3}i)^{1/3}$
$d)\quad \left(z^{1/2}\right)^{1/3} \qquad e)\quad \left(z^{5/2}\right)^{2/5} \qquad f)\quad \ln(\ln i)$

*Exercise 7

Show that the cross-ratio is an invariant of fractional transformation, i.e., that

$$\frac{(w_1-w_2)(w_3-w_4)}{(w_1-w_3)(w_2-w_4)} = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_3)(z_2-z_4)}$$

when $w = \frac{az+b}{cz+d}$.

Exercise 8(CKP, page 19, problem 1)

Use the cross-ratio to obtain a mapping which transforms the upper-half z plane into the interior of the unit circle in the w plane. Sketch the w images of vaious points and curves in the z plane, and vice versa. (*Hint*: Replace z_1, w_1 by z, w; set z_2, z_3, z_4 equal to $-1, 0, 1$, etc. Or use point at infinity.)

*Exercise 9(CKP, page 24, problem 13a)

Discuss the branch cut and Riemann-surface structure for the following function

$$g(z) = \sqrt{1+\sqrt{z}}.$$