Physics 503: Methods of Mathematical Physics

Read: **CKP** chapter 4, sections 4-1 – 4-5.

"**CKP**" refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 4

Exercise 1

Find the nature of each singularity (including the point at infinity) of each of the following functions.

$a)\ \frac{e^{1/(z-1)}}{e^z-1},\qquad b)\ \frac{1}{(\sin z+\cos z)^3},\qquad c)\ z^2e^{-z},\qquad d)\ \frac{z^{2n}}{(1+z^n)^2},\qquad e)\ \frac{\ln(z-1)}{(z+1)^2}.$

Evaluate the residues at each isolated singularity. Always include the point at ∞ in your considerations.

The following problems are all two-dimensional. Charge means 2d charge etc.

Exercise 2

Find the equipotentials, streamlines, and vector intensity ($\vec{\nabla}\phi$) for the field whose complex potential $\Omega = \frac{1}{z^2}$.

Exercise 3

The equipotential lines of a fluid are the circles $x^2 + y^2 = 2ax$ (for any a). Find the ratio of the magnitudes of the intensity of the field at the points $(2a, 0)$ and (a, a).

Exercise 4: Method of images

Find the force acting on electric charge q (force is equal $-q\vec{\nabla}\phi$) inside the right angle corner surrounded by metal (see figure).

*Exercise 5: Streamlining the parabola

Find the velocity field for the flow streamlining the parabolic surface given by $y^2 = 2px$ with the given velocity at infinity $v(x \to -\infty) = v_\infty$ (see figure).

Hint: use the mapping $\zeta = \sqrt{z - \frac{p}{2}}$.

Figure 1: Figure to exercise 4

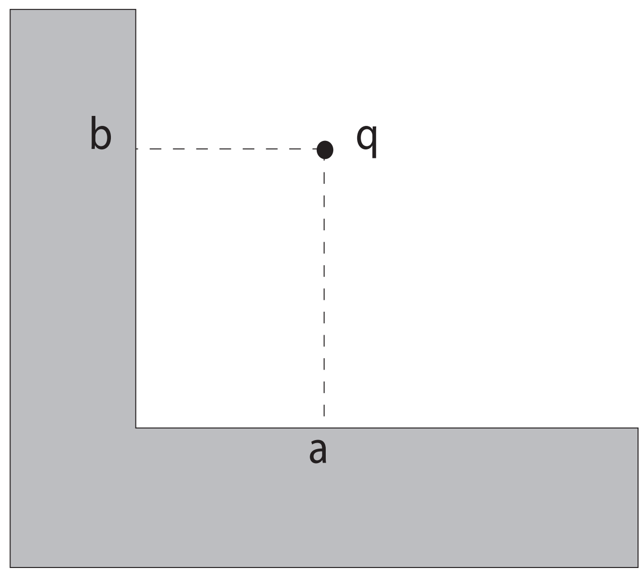


Figure 2: Figure to exercise 5

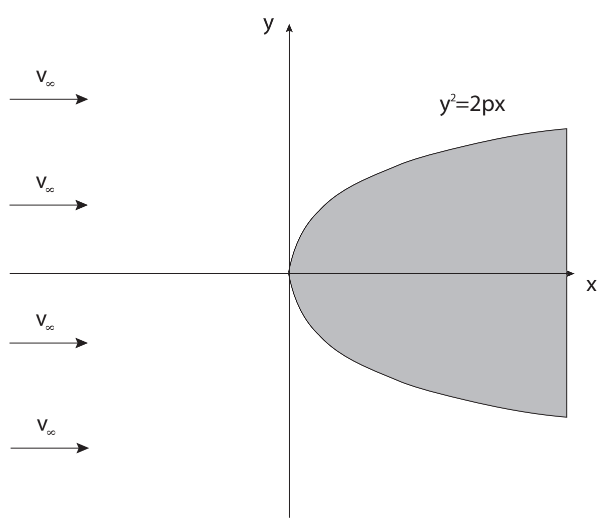
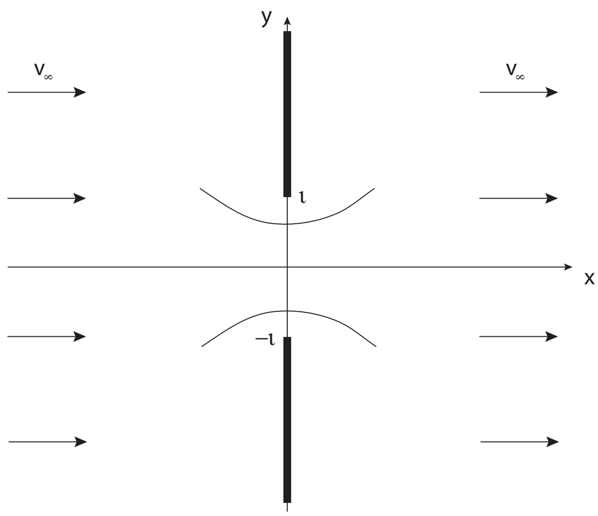


Figure 3: Figure to exercise 8



Exercise 6 (CKP, page 159, problem 1)

a) Show that Joukowsky transformation $z = \frac{1}{2}a\left(\zeta + \frac{1}{\zeta}\right)$ can be expressed in the useful alternative form $\frac{z-a}{z+a} = \left(\frac{\zeta-1}{\zeta+1}\right)^2$.

b) Show that the transformation doubles angles at the points $\zeta = \pm 1$.

Exercise 7

Using Joukowsky transformation find how a uniform electric field $\vec{E} = -\vec{\nabla}\phi = const$ is perturbed by metal circle of unit radius.

Exercise 8

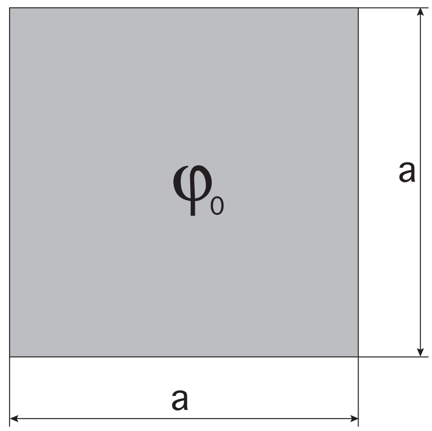
Find the complex potential for the plane flow of a fluid flowing from the left half-plane into the right half-plane through an aperture in the imaginary axis between the points $-i$ and i (see figure). The net flow across the aperture is Q $(= \int_{-1}^{1} \phi_x dy)$.

Hint: Consider $\zeta = \ln(z + \sqrt{z^2+1})$ with properly chosen branching lines.

*Exercise 9

Find the electric field (complex intensity $\Omega'(z)$) produced by a metal square of size a (see figure) and charge q at distanses $|z| \gg a$. Keep the next to a leading term in

Figure 4: Figure to exercise 9



the expansion in $a/|z|$.

Hint: Use the Schwartz-Christoffel mapping to the exterior of the unit circle. Expand in $a/|z|$.