9/4/2008. **Due:** Tuesday, Sept. 16, 2008 (in class).

Physics 503: Methods of Mathematical Physics

Read: CKP chapter 1

"**CKP**" refers to Carrier, Krook, and Pearson book.
Problems with stars are not for credit and will NOT be graded.

Homework 1

Exercise 1

Calculate real and imaginary parts of the following complex numbers:

a) $2 + \sqrt{17}i$ b) $(\sqrt{2} - \sqrt{3}i)^2$ c) $\frac{2+3i}{5-i}$

d) $\left(\frac{1-i}{5}\right)^{15}$ e) $(2+5i)^{20}$ f) $\left(\frac{1+i}{5}\right)^{17} + (2+5i)^{30}$

Exercise 2

Find $\sin(5\theta)$ in terms of $\sin\theta$ using de Moivre's formula and identity $\sin^2\theta = 1 - \cos^2\theta$.

Exercise 3

Consider the sequence defined by recurrent relation and initial conditions.

$$\begin{aligned}
F_k &= 2F_{k-1} - 2F_{k-2}, \\
F_0 &= 1, \\
F_1 &= 5.
\end{aligned}$$

Write down the geometric sequence ansatz and find the roots of the corresponding quadratic equation. Write down the formula for F_k using initial conditions. Analyze the result using de Moivre's formula. What is the value (order of magnitude) of F_{103}?

*Exercise 4(CKP, page 5, problem 6)

Find the value of the following sum in a compact form

$$1 + r\cos\theta + r^2\cos 2\theta + \ldots + r^n\cos n\theta.$$

Hint: Write it as a real part of a complex geometric sequence using de Moivre's formula, sum it up, and find the real part of the result.

*Exercise 5

Calculate real and imaginary parts of the principal value of the following complex numbers:

$$a) \quad \ln(1 + \sqrt{3}i) \qquad\qquad b) \quad \ln(-5)$$
$$c) \quad 2^{-i} \qquad\qquad\qquad d) \quad (1 - 3i)^{1/3}$$

Exercise 6

Find "all" multiple values (in arbitrary form) of the following expressions

$$a) \quad \ln(1 + \sqrt{3}i) \qquad b) \quad 1^{3/5} \qquad\qquad c) \quad (1 + \sqrt{3}i)^{1/3}$$
$$d) \quad \left(z^{1/2}\right)^{1/3} \qquad\quad e) \quad \left(z^{5/2}\right)^{2/5} \qquad f) \quad \ln(\ln i)$$

*Exercise 7

Show that the cross-ratio is an invariant of fractional transformation, i.e., that

$$\frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_3)(w_2 - w_4)} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_3)(z_2 - z_4)}$$

when $w = \frac{az+b}{cz+d}$.

Exercise 8(CKP, page 19, problem 1)

Use the cross-ratio to obtain a mapping which transforms the upper-half z plane into the interior of the unit circle in the w plane. Sketch the w images of vaious points and curves in the z plane, and vice versa. (*Hint*: Replace z_1, w_1 by z, w; set z_2, z_3, z_4 equal to $-1, 0, 1$, etc. Or use point at infinity.)

*Exercise 9(CKP, page 24, problem 13a)

Discuss the branch cut and Riemann-surface structure for the following function

$$g(z) = \sqrt{1 + \sqrt{z}}.$$