10/02/2008. **Due:** Tuesday, October 28, 2008 (in class).

Physics 503: Methods of Mathematical Physics

Read: CKP chapter 2, sections 2-6 – 2-8.
chapter 3, sections 3-1, 3-2.

"**CKP**" refers to Carrier, Krook, and Pearson book.
"**FS**" refers to Fuchs and Shabat book.
Problems with stars are not for credit and will NOT be graded.

Homework 3

Exercise 1

Find the nature of each singularity (including the point at infinity) of each of the following functions.

a) $\frac{e^{1/(z-1)}}{e^z-1}$, *b)* $\frac{1}{(\sin z+\cos z)^3}$, *c)* $z^2 e^{-z}$, *d)* $\frac{z^{2n}}{(1+z^n)^2}$, *e)* $\frac{\ln(z-1)}{(z+1)^2}$.

Evaluate the residues at each isolated singularity. Always include the point at ∞ in your considerations.

Exercise 2

Evaluate

$$I = \int_0^\infty \frac{dx}{1+x^{2007}}.$$

Exercise 3

Evaluate

$$I = \int_0^\infty \frac{dx}{x^3+x^2+x+1}.$$

Use "logarithm trick".

Exercise 4 (CKP, page 82, problem 1)

Evaluate (using Jordan's lemma where necessary)

$$I = \int_0^\infty \frac{x \sin x}{a^2+x^2}\, dx.$$

Exercise 5 (CKP, page 89, problem 8)

Evaluate

$$I = \int_0^{2\pi} \ln(a + b\cos\theta)\, d\theta$$

for $a > b > 0$.

Exercise 6 (CKP, page 90, problem 12)

Show that

$$(a) \qquad \int_0^{2\pi} e^{\cos\theta} \cos(n\theta - \sin\theta)\, d\theta = \frac{2\pi}{n!}$$

$$(b) \qquad \int_0^{2\pi} e^{\cos\theta} \sin(n\theta - \sin\theta)\, d\theta = 0$$

Exercise 7 (CKP, page 90, problem 14)

Evaluate

$$I = \int_0^{2\pi} \frac{x \sin x}{1 - 2\alpha\cos x + \alpha^2}\, dx$$

for α real and for each of the two cases $|\alpha| < 1$, $|\alpha| > 1$.

Exercise 8 (CKP, page 90, problem 16)

Evaluate

$$a) \qquad I = \int_0^{\infty} \frac{\ln(1 + x^2)}{1 + x^2}\, dx$$

$$b) \qquad I = \int_0^{\infty} \frac{(\ln x)^2}{1 + x^2}\, dx$$

Exercise 9 (FS 81.1cg)

Evaluate the following integrals

$$I = \int_{-\infty}^{\infty} \frac{\cos ax}{1 + x^4}\, dx, \qquad\qquad I = \int_{-1}^{1} \frac{dx}{[(1 - x)(1 + x)^2]^{1/3}}.$$

*Exercise 10

Show that

$$a) \qquad \cot z = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - \pi^2 n^2}$$

$$b) \qquad \frac{1}{\sin^2 z} = \sum_{n=-\infty}^{+\infty} \frac{1}{(z - \pi n)^2}$$