Homework 10

Reading

LL 111, **EM** 4.6, **KKL** 3.2.

Problem 1

The Hamiltonian of charged particle in uniform magnetic field B is given by

$$H = \frac{1}{2m}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)^2 = \frac{m\boldsymbol{v}^2}{2}, \tag{1}$$

where $\partial_x A_y - \partial_y A_x = B$. A classical particle moves according to

$$\begin{aligned} v_y &= -\omega_B(x - x_0), \\ v_x &= \omega_B(y - y_0), \end{aligned} \tag{2}$$

where $\omega_B = eB/(mc)$ is the cyclotron frequency and x_0, y_0 are the coordinates of the center of the circular orbit. The radius of the orbit R can be determined from $\omega_B^2 R = \boldsymbol{v}^2/R$.

Let us define the QM operators of coordinates of the center of the orbit x_0, y_0, the square of the radius-vector of the center r_0^2 and the square of the radius of the orbit R^2 respectively as

$$x_0 = x + \frac{1}{\omega_B} v_y, \tag{3}$$

$$y_0 = y - \frac{1}{\omega_B} v_x, \tag{4}$$

$$r_0^2 = x_0^2 + y_0^2, \tag{5}$$

$$R^2 = \frac{1}{\omega_B^2}(v_x^2 + v_y^2). \tag{6}$$

Find commutators of these operators with each other. Are the corresponding observables compatible?

Problem 2

Find the Heisenberg equations of motion for the operators (3-6) introduced in the previous problem. Compare the results with classical analogues and make conclusions.

Problem 3

A Hamiltonian of a particle on a ring of radius R in the presence of magnetic flux has a form

$$H = \frac{L^2}{2mR^2}, \tag{7}$$

where L is an operator of an angular momentum $L = -i\hbar\partial_\phi - \frac{e}{c}\frac{A_\phi}{R}$, where A_ϕ is the component of the vector potential along the ring. One can choose the gauge in which $A_\phi = const$.

a) Relate A_ϕ to the total flux of magnetic field through the ring Φ.

b) Using periodic boundary conditions $\psi(\phi) = \psi(\phi + 2\pi)$ find all eigenenergies and eigenfunctions of the Hamiltonian.

c) Plot the dependence of the energy $E_0(\Phi)$ of the ground state (the lowest energy eigenstate) on Φ.

d) Find the magnetic susceptibility of the system in the ground state as $\chi = -\partial^2 E_0/(\partial\Phi)^2$ at $\Phi = 0$.

Problem 4

A two-dimensional oscillator is put into a circular box with impenetrable walls and radius R. Estimate (e.g., using the variational approach) the energy of the ground state of this system for the following cases.

a) $R \ll \lambda$, where λ is an oscillator length;

b) $R \gg \lambda$;

c) $R = 2\lambda$.