# Homework 11

Problems with stars are not for credit and will NOT be graded.

## Reading

**LL** 111, 26, **EM** 4.6, 11.1, **KKL** 3.2, 4.11.

## Problem 1

Find the ground state energy and the wave function of a 2d particle in a constant magnetic field $B$ and in a **strong** external harmonic potential.

$$H = \frac{1}{2m}\left(-i\hbar\boldsymbol{\nabla} - \frac{e}{c}\boldsymbol{A}\right)^2 + \frac{m\omega_0^2}{2}(x^2+y^2).$$

What is the expectation value $\langle x^2+y^2\rangle$ in the ground state? Express the result in terms of magnetic length $l$ and oscillator length $\lambda$.

*Hint:* Without magnetic field the angular momentum of the ground state is zero and this will not be changed in magnetic field as the harmonic potential is strong.

## Problem 2

What is the magnetic moment of the particle in the ground state for the system in the previous problem? How does it behave at small $B$? What is the magnetic susceptibility of the ground state of a 2d oscillator at $B = 0$?

*Hint:* Magnetic moment is given by $M = -\partial E/\partial B$ and the susceptibility is defined as $\chi = \partial M/\partial B$.

## Problem 3

This problem is a toy problem illustrating some physics of an Integer Quantum Hall Effect (IQHE).

Consider the electron gas confined to the two dimensional $xy$ plane. Let us neglect the interaction between electrons. The Hamiltonian of a single particle is given by

$$H = \frac{1}{2m}\left(-i\hbar\boldsymbol{\nabla} - \frac{e}{c}\boldsymbol{A}\right)^2 + V(x,y),$$

where $\boldsymbol{A}$ is a vector potential of magnetic field and $V(x,y)$ is an additional electrostatic (confining) potential. For simplicity we will take the confining potential to be a one-dimensional harmonic potential $V = \frac{1}{2}m\omega_0^2 y^2$.

a. For the constant magnetic field $B$ using the Landau gauge $A_x = -By$, $A_y = 0$ and a separation of variables $\psi(x,y) = \psi_k(y)e^{-ikx}$ write down the stationary Schrödinger equation for $\psi_k(y)$.

b. Identify the obtained equation as the one for harmonic oscillator and find the energy levels $E_{n,k}$ with $n = 0, 1, 2, \ldots$. The levels at given $n$ are said to belong to the same *Landau level*.

c. Let us assume that the chemical potential $\mu$ is such that Landau levels with $n > 0$ are empty (i.e., $E_{n,k} > \mu$ for $n > 0$). Then the only occupied states are the ones with $n = 0$. What are the maximal and minimal values of $k$ of occupied levels?

d. What are positions (in $y$ direction) of those occupied levels?

*e. The states with maximal and minimal $k$ are called the edge states of IQHE. Find the velocity of corresponding boundary excitations.

## Problem 4

Calculate the following commutators:

$$a)\ [L_i, \boldsymbol{p}^2], \qquad b)\ [L_i, (\boldsymbol{p}\cdot\boldsymbol{r})], \qquad c)\ [L_i, \alpha\boldsymbol{p} + \beta\boldsymbol{r}], \qquad d)\ [L_i, (\boldsymbol{p}\cdot\boldsymbol{r})\boldsymbol{r}], \qquad e)\ [L_i, x_k x_l].$$

Here operators $\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{L}$ are canonical operators of radius-vector, momentum and angular momentum of a particle in three dimensions respectively. The subscripts $i, k, l$ can take values $x, y, z$ and $\alpha, \beta$ are some constants. Do you notice any similarities between some of the commutators?