Homework 11

Problems with stars are not for credit and will NOT be graded.

Reading

LL 111, 26, EM 4.6, 11.1, KKL 3.2, 4.11.

Problem 1

Find the ground state energy and the wave function of a 2d particle in a constant magnetic field \( B \) and in a strong external harmonic potential.

\[
H = \frac{1}{2m} \left(-i\hbar \nabla - \frac{e}{c} A\right)^2 + \frac{m\omega_0^2}{2}(x^2 + y^2).
\]

What is the expectation value \( \langle x^2 + y^2 \rangle \) in the ground state? Express the result in terms of magnetic length \( l \) and oscillator length \( \lambda \).

*Hint:* Without magnetic field the angular momentum of the ground state is zero and this will not be changed in magnetic field as the harmonic potential is strong.

Problem 2

What is the magnetic moment of the particle in the ground state for the system in the previous problem? How does it behave at small \( B \)? What is the magnetic susceptibility of the ground state of a 2d oscillator at \( B = 0 \)?

*Hint:* Magnetic moment is given by \( M = -\partial E/\partial B \) and the susceptibility is defined as \( \chi = \partial M/\partial B \).

Problem 3

This problem is a toy problem illustrating some physics of an Integer Quantum Hall Effect (IQHE).

Consider the electron gas confined to the two dimensional \( xy \) plane. Let us neglect the interaction between electrons. The Hamiltonian of a single particle is given by

\[
H = \frac{1}{2m} \left(-i\hbar \nabla - \frac{e}{c} A\right)^2 + V(x, y),
\]

where \( A \) is a vector potential of magnetic field and \( V(x, y) \) is an additional electrostatic (confining) potential. For simplicity we will take the confining potential to be a one-dimensional harmonic potential \( V = \frac{1}{2}m\omega_0^2 y^2 \).

a. For the constant magnetic field \( B \) using the Landau gauge \( A_x = -By, A_y = 0 \) and a separation of variables \( \psi(x, y) = \psi_k(y)e^{-ikx} \) write down the stationary Schrödinger equation for \( \psi_k(y) \).

b. Identify the obtained equation as the one for harmonic oscillator and find the energy levels \( E_{n,k} \) with \( n = 0, 1, 2, \ldots \). The levels at given \( n \) are said to belong to the same Landau level.
c. Let us assume that the chemical potential $\mu$ is such that Landau levels with $n > 0$ are empty (i.e., $E_{n,k} > \mu$ for $n > 0$). Then the only occupied states are the ones with $n = 0$. What are the maximal and minimal values of $k$ of occupied levels?

d. What are positions (in $y$ direction) of those occupied levels?

*e. The states with maximal and minimal $k$ are called the edge states of IQHE. Find the velocity of corresponding boundary excitations.

**Problem 4**

Calculate the following commutators:

$$a) [L_{i,\alpha}, p^2], \quad b) [L_{i,\alpha}, (p \cdot r)], \quad c) [L_{i,\alpha}, \beta p + \beta r], \quad d) [L_{i,\alpha}, (p \cdot r)r], \quad e) [L_{i,\alpha}, x_i x_k].$$

Here operators $r, p, L$ are canonical operators of radius-vector, momentum and angular momentum of a particle in three dimensions respectively. The subscripts $i, k, l$ can take values $x, y, z$ and $\alpha, \beta$ are some constants. Do you notice any similarities between some of the commutators?