

Homework 5

Problem 1

a) Find the potential profile $U(x)$ for which the following wavefunction,

$$\Psi(x, t) = C e^{-a|x| - ibt}$$

(with real and constant $a > 0$ and b), satisfies the Schrödinger equation for a particle with mass m .

b) Normalize the wavefunction to $P = 1$, find $\langle x \rangle$, $\langle p_x \rangle$, δx , and δp_x , and compare the product $\delta x \delta p_x$ with Heisenberg's uncertainty relation.

Problem 2

Find the transmission coefficient $T(E)$ as a function of the energy $E > 0$ of the particle scattered by a potential

$$U(x) = -W\delta(x).$$

Problem 3

Find the number of bound states as a function of the parameter $\xi = mWa/\hbar^2$ for the particle moving in a potential $U(x)$ of the form

$$U(x) = \begin{cases} \infty, & x < 0, \\ -W\delta(x - a), & x > 0. \end{cases}$$

Problem 4

Find the values of energies at which particles are not reflected by the potential barrier

$$U(x) = W[\delta(x) + \delta(x - a)].$$