Homework 7

Problem 1

For a 1D harmonic oscillator with mass m and frequency ω_0, calculate:

(i) all matrix elements $\langle n|\,\hat{x}^3\,|n'\rangle$, and

(ii) diagonal matrix elements $\langle n|\,\hat{x}^4\,|n\rangle$,

where $|n\rangle$ are Fock states.

Problem 2

Consider a quantum harmonic oscillator with the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2. \tag{1}$$

Let us denote $|n\rangle$ with $n = 0, 1, 2, \ldots$ the normalized eigenstates of this Hamiltonian.

a) Find the matrix elements $\langle 5|x|6\rangle$, $\langle 4|x|6\rangle$, $\langle 5|x^2|7\rangle$, $\langle 11|p + m\omega x|11\rangle$.

b) What is the time evolution of the matrix element $\langle 5|x|6\rangle$?

c) What is the expectation value of the observable $h = p^2 + m^2\Omega^2 x^2$ in the eigenstate $|n\rangle$ of the harmonic oscillator (1)?

Problem 3

Consider a particle moving in 1d potential (Morse potential)

$$H = \frac{p^2}{2m} + A\Big(e^{-2\alpha x} - 2e^{-\alpha x}\Big), \tag{2}$$

where A and $\alpha > 0$ are some constants.

a) Consider the operator $B = -\partial_x + Ce^{-\alpha x} - D$ and its Hermitian conjugate $B^\dagger$. Here C and D are some constants. Find the Hamiltonians $H_- = B^\dagger B$ and $H_+ = BB^\dagger$. Find the values of constants C and D so that H_- is equal to H up to an additive constant.

b) Using the results of a) and solving the equation $B\psi = 0$ find the ground state energy and the ground state wave function for (2).

Problem 4

Using trial wave functions of the form a) $\Psi(x) = Axe^{-\alpha x}$ and b) $\Psi(x) = Bxe^{-\alpha x^2/2}$, where α is a variational parameter find an approximate energy of the ground state of the particle moving in the potential

$$U(x) = \begin{cases} kx, & x > 0 \quad (k > 0), \\ +\infty, & x < 0. \end{cases} \tag{3}$$

Compare with the exact result $E_0 \approx 2.338(\hbar^2 k^2/2m)^{1/3}$.