Homework 8

Reading

LL 23, **EM** Ch 5, **JJS** 2.2.

Problem 1

A linear harmonic oscillator is exposed to a spatially constant force. That is the Hamiltonian of this oscillator has a form

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} - Fx. \tag{1}$$

The oscillator is in its ground state. At time $t = 0$ the force is suddenly removed. Compute the transition probabilities to the excited states of the oscillator. Use the generating function for Hermite polynomials to obtain a general formula.

Problem 2

a) Find the eigenstates and eigenenergies of the particle in the following potential

$$V(x) = \begin{cases} \frac{m\omega^2 x^2}{2}, & \text{for } x > 0, \\ +\infty, & \text{for } x < 0. \end{cases} \tag{2}$$

b) A particle is in the ground state of the harmonic potential $\frac{m\omega^2 x^2}{2}$. Suddenly, at time $t = 0$ an impenetrable partition is inserted at the point $x = 0$. Find the time dynamics of the state. Namely, what is the wave function of the particle at time t?

c) At what time one should remove the partition to have again a particle in the ground state of the oscillator potential?

Hints: In part a) think about parity symmetry of eigenfunctions of linear oscillator problem. In part b) use the property of sudden perturbations. You can also use table values of integrals with Hermite polynomials.

Problem 3

Consider a tight binding model on a triangle given by a Hamiltonian

$$H = -W \sum_{n=1}^{3} \Big(|n\rangle \langle n+1| + |n+1\rangle \langle n| \Big). \tag{3}$$

a) Find the evolution operator $U(t)$ for this model as an explicit 3×3 matrix.

b) Using the found evolution operator obtain the state of the system at time t if at $t = 0$ it was in the state $|1\rangle$.

Hint: It is convenient to use plane wave eigenstates $|p\rangle$.