Quantum Mechanics Recitation #1 Overview

Tin Sulejmanpašić
Stony Brook University, e-mail: tin2019@gmail.com

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1. Let A, B be operators, such that $[A, B] = c$, where c is a complex number. Show that

$$[A, f(B)] = f'(B)[A, B] \ . \tag{1}$$

where $f(x)$ is a smooth, continuous and infinitely differentiable function (i.e. can be defined through its Taylor series).

Consider the special cases when $A = p$ and $B = x$ and $A = x$ and $B = p$

2. Compute

$$[x, T_a] \tag{2}$$

where x is the position operator and T_a is the translation operator defined as

$$T_a = e^{-\frac{i}{\hbar}ap} \ , \tag{3}$$

3. Diagonalize

$$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{pmatrix} \tag{4}$$

4. Let A be a Hermitian operator, and let $\{|a\rangle\}$ a complete basis of eigenkets of that operator.

   (a) Show that if there are no degeneracies that this basis is orthogonal.

   (b) Show that if there are degeneracies, we can always construct a orthogonal basis

5. Let

$$A = \sum_{i=1}^{N} A_i \qquad B = \sum_{i=1}^{N} B_i \ , \tag{5}$$

where A_i, B_j are operators for which

$$[A_i, A_j] = 0 \ , \qquad [A_i, B_j] = \delta_{ij} \ . \tag{6}$$

Compute $[A, C]$ where

$$C = AB \ . \tag{7}$$

6. Let A, B be observables. Suppose the simultanious eigenkets of A and B $\{|a', b'\rangle\}$ form a complete orthonormal set of base kets. Can we always conclude that

$$[A, B] = 0 \ . \tag{8}$$

7. Let A_1, A_2 be operators such that $[A_1, A_2] \neq 0$, but for which $[A_1, H] = [A_2, H] = 0$. Show that in general the energy eigenstates are degenerate.

8. Consider two operators in a particular basis

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} , \qquad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix} , \tag{9}$$

   (a) A obviously has degenerate spectra. What about B

   (b) Show that $[A, B] = 0$

   (c) Find simultaneous eigenkets of A and B