Homework 8

Reading

LL 23, **EM** Ch 5, **JJS** 2.2.

Problem 1

Using dimensional analysis estimate the energy of the ground state of the particle moving in the potential

$$U(x) = \begin{cases} kx^3, & x > 0 \quad (k > 0), \\ +\infty, & x < 0. \end{cases} \tag{1}$$

Problem 2

For a 1D harmonic oscillator with mass m and frequency ω_0, calculate:

(i) The expectation value of the potential energy in the eigenstate $|n\rangle$,

(ii) all matrix elements $\langle n| \hat{x}^3 |n'\rangle$, and

(iii) diagonal matrix elements $\langle n| \hat{x}^4 |n\rangle$,

where $|n\rangle$ are Fock states.

Note: Solve this problem using ladder operators.

Problem 3

Consider a particle moving in 1d potential (Morse potential)

$$H = \frac{p^2}{2m} + A\Big(e^{-2\alpha x} - 2e^{-\alpha x}\Big), \tag{2}$$

where A and $\alpha > 0$ are some constants.

a) Consider the operator $B = -\partial_x + Ce^{-\alpha x} - D$ and its Hermitian conjugate $B^\dagger$. Here C and D are some constants. Find the values of constants C and D so that $H = B^\dagger B + const$.

b) Using the results of a) and solving the equation $B\psi = 0$ find the ground state energy and the ground state wave function for (1).

Problem 4

Using trial wave functions of the form a) $\Psi(x) = Axe^{-\alpha x}$ and b) $\Psi(x) = Bxe^{-\alpha x^2/2}$, where α is a variational parameter find an approximate energy of the ground state of the particle moving in the potential

$$U(x) = \begin{cases} kx, & x > 0 \quad (k > 0), \\ +\infty, & x < 0. \end{cases} \tag{3}$$

Compare with the exact result $E_0 \approx 2.338(\hbar^2 k^2/2m)^{1/3}$.

Problem 5

A linear harmonic oscillator is exposed to a spatially constant force. That is the Hamiltonian of this oscillator has a form

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} - Fx. \tag{4}$$

The oscillator is in its ground state. At time $t = 0$ the force is suddenly removed. Compute the transition probabilities to the excited states of the oscillator. Use the generating function for Hermite polynomials to obtain a general formula.

Problem 6

a) Find the eigenstates and eigenenergies of the particle in the following potential

$$V(x) = \begin{cases} \frac{m\omega^2 x^2}{2}, & \text{for } x > 0, \\ +\infty, & \text{for } x < 0. \end{cases} \tag{5}$$

b) A particle is in the ground state of the harmonic potential $\frac{m\omega^2 x^2}{2}$. Suddenly, at time $t = 0$ an impenetrable partition is inserted at the point $x = 0$. Find the time dynamics of the state. Namely, what is the wave function of the particle at time t?

c) At what time one should remove the partition to have again a particle in the ground state of the oscillator potential?

Hints: In part a) think about parity symmetry of eigenfunctions of linear oscillator problem. In part b) use the property of sudden perturbations. You can also use table values of integrals with Hermite polynomials.

Problem 7

Consider a tight binding model on a triangle given by a Hamiltonian

$$H = -W \sum_{n=1}^{3} \Big(|n\rangle \langle n+1| + |n+1\rangle \langle n| \Big). \tag{6}$$

a) Find the evolution operator $U(t)$ for this model as an explicit 3×3 matrix.

b) Using the found evolution operator obtain the state of the system at time t if at $t = 0$ it was in the state $|1\rangle$.

Hint: It is convenient to use plane wave eigenstates $|p\rangle$.