Physics 501: Classical Mechanics

Read: LL 1-5, G 1,2
Special attention: LL problems after ch. I

Homework 1

Exercise 1 (G 2.1)
Using variational calculus prove that the shortest distance between two points in space is a straight line.

Exercise 2 (G 1.15)
Let \( q_1, \ldots, q_n \) be a set of independent generalized coordinates for a system of \( n \) degrees of freedom, with a Lagrangian \( L(q, \dot{q}, t) \). Suppose we transform to another set of independent coordinates \( s_1, \ldots, s_n \) by means of transformation equations

\[
q_i = q_i(s_1, \ldots, s_n, t), \quad i = 1, \ldots, n.
\]

(Such a transformation is called a point transformation.) Show that if the Lagrangian function is expressed as a function of \( s_j, \dot{s}_j, \) and \( t \) through the equations of transformation, then \( L \) satisfies Lagrange’s equations with respect to the \( s \) coordinates:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{s}_j} \right) - \left( \frac{\partial L}{\partial s_j} \right) = 0.
\]

In other words, the form of Lagrange’s equations is invariant under a point transformation.

Remark: Do not use variational principle.

Exercise 3
For each of the systems listed below:
- introduce convenient generalized coordinates \( q_j \),
- write down the Lagrangian \( L \) as a function of \( q_j, \dot{q}_j \) and (if appropriate) time,
- write down the Lagrangian equations of motion.

a) A particle of the mass \( m \) sliding without friction on a heavy wedge of angle \( \alpha \) and mass \( M \) that can move without friction on a smooth horizontal surface.
b) A simple pendulum of the mass $m$ whose point of support moves vertically according to the prescribed law $y = y_0(t)$. Pendulum motion is constrained to a vertical plane.

c) A particle of the mass $m$ confined to the surface of the sphere (but freely moving along the surface) of radius $R$ placed in a uniform gravitational field (acceleration $g$).

d) A bead of the mass $m$ moving without friction along the rigid wire of elliptic shape $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

**Exercise 4**

Using variational calculus with Lagrange multiplier, find the shape of the rope of the length $l$ suspended between two equally high points. The horizontal distance between points is $2a$. 