# Physics 501: Classical Mechanics

*Read:* **LL** 1-5, **G** 1,2
*Special attention:* **LL** problems after ch. I

## Homework 1

### Exercise 1 (G 2.1)

Using variational calculus prove that the shortest distance between two points in space is a straight line.

### Exercise 2 (G 1.15)

Let $q_1, \ldots, q_n$ be a set of independent generalized coordinates for a system of $n$ degrees of freedom, with a Lagrangian $L(q, \dot{q}, t)$. Suppose we transform to another set of independent coordinates $s_1, \ldots, s_n$ by means of transformation equations

$$q_i = q_i(s_1, \ldots, s_n, t), \qquad i = 1, \ldots, n.$$

(Such a transformation is called a *point transformation.*) Show that if the Lagrangian function is expressed as a function of $s_j$, $\dot{s}_j$, and $t$ through the equations of transformation, then $L$ satisfies Lagrange's equations with respect to the $s$ coordinates:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}_j}\right) - \left(\frac{\partial L}{\partial s_j}\right) = 0.$$

In other words, the form of Lagrange's equations is invariant under a point transformation.

*Remark:* Do not use variational principle.

### Exercise 3

For each of the systems listed below:

-introduce convenient generalized coordinates $q_j$,
-write down the Lagrangian $L$ as a function of $q_j$, $\dot{q}_j$ and (if appropriate) time,
-write down the Lagrangian equations of motion.

a) A particle of the mass $m$ sliding without friction on a heavy wedge of angle $\alpha$ and mass $M$ that can move without friction on a smooth horizontal surface.

b) A simple pendulum of the mass $m$ whose point of support moves vertically according to the prescribed law $y = y_0(t)$. Pendulum motion is constrained to a vertical plane.

c) A particle of the mass $m$ confined to the surface of the sphere (but freely moving along the surface) of radius $R$ placed in a uniform gravitational field (acceleration $g$).

d) A bead of the mass $m$ moving without friction along the rigid wire of elliptic shape $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

## Exercise 4

Using variational calculus with Lagrange multiplier, find the shape of the rope of the length $l$ suspended between two equally high points. The horizontal distance between points is $2a$.