11/16/2011 **Due:** Monday, Dec. 5, 2011 (in class).

Physics 501: Classical Mechanics

Read: **LL** 27-30; **JS** 6, 7; **G** 11, 12.

Homework 10

Exercise 1 (JS 6.6)

Use the results of the secular perturbation theory treatment of the quartic oscillator to obtain the first order correction for the frequency of a simple pendulum whose amplitude is Θ.

Exercise 2

Find the 2-cycle for the logistic equation $x_{n+1} = a x_n (1 - x_n)$. Analyze the stability of this cycle (find the range of a where it is stable).

Exercise 3

The standard map (corresponding to a kicked rotator) is given by

$$\begin{aligned} \phi_{n+1} &= (\phi_n + J_{n+1}) \bmod 2\pi, \\ J_{n+1} &= \epsilon \sin \phi_n + J_n. \end{aligned}$$

Here ϕ and J are the angle and the velocity of the rotator, and ϵ is the strength of kicks (control parameter). Using Maple, Mathematica, or similar software draw Poincare maps (trajectories in $\phi - J$ plane) for this system for several values of ϵ, e.g., $\epsilon = 0, 0.8, 5$.

Exercise 4

Consider a periodically driven pendulum described by

$$\ddot{\phi} + \omega_0^2 \left[1 + h \cos(2\omega_0 + \epsilon)t\right] \sin \phi = 0.$$

Here $h \ll 1$.

a) Consider a non-driven pendulum ($h = 0$). Due to the nonlinearity of the problem the frequency of oscillations is amplitude dependent. Find the correction to the frequency of oscillations for the finite (but small) amplitude of oscillations $\phi_0 \ll 1$.

b) Assume now that $h \ll 1$ but is not zero. At small ϵ the parametric resonance occurs. State the condition (on ϵ) for a parametric resonance.

c) Let us now assume that $\epsilon = 0$. Parametric resonance occurs and amplitude of oscillations grows with time. Due to the nonlinearity of the problem the basic frequency of oscillations changes with the amplitude of oscillations and at some point the parametric resonance condition will be violated. Estimate the maximal (saturation) amplitude ϕ_{max} of oscillations as a function of $h \ll 1$.