

Physics 501: Classical Mechanics

Read: LL 27-30; JS 7; G 11; LL7 I,II.

Homework 11

Exercise 1

Determine the deformation of a long rod (with length L) rotating with frequency Ω around its end (the axis of rotation is orthogonal to the rod).

Exercise 2

Calculate the deformation law $h(x)$ for a thin, heavy, elastic board which is weakly bent by the Earth gravity (see Figure 1). Assume that the Young's modulus E , the mass of the board, and its dimensions are known. How does the maximal deflection of the board scale with L ?

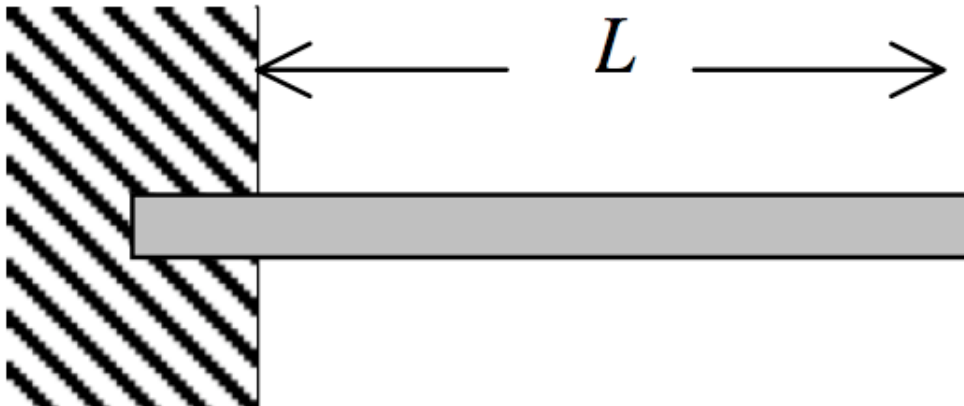


Figure 1: To Exercise 5.

Exercise 3

Find the stationary shape of the surface of an incompressible fluid rotated around the vertical axis with a constant angular velocity Ω .

Exercise 4 (KdV equation)

Consider the continuous system which is defined by a Hamiltonian (given as a functional of the field $u(x)$)

$$H[u] = \int dx \frac{1}{6} [u^3 + 3u_x^2]$$

and by Poisson bracket

$$\{u(x), u(y)\} = \partial_x \delta(x - y).$$

a) Derive the equation of motion for the field $u(x, t)$.

b) Assume that the system has periodic boundary conditions. Writing $u(x) = \sum_p u_p e^{ipx}$ find the Poisson bracket $\{u_p, u_q\}$. Suggest the canonically conjugated coordinates and momenta for the dynamical system.