9/12/2011. **Due:** Monday, Sep. 19, 2011 (in class).

Physics 501: Classical Mechanics

Read: **LL** 5-10; **G** 2.6; **JS** 3.2.2
Special attention: **LL** problems in ch. II

Homework 2

Exercise 1

Show that the Lagrangian of a system of particles

$$L = \sum_{k=1}^{N} \frac{m_k v_k^2}{2} - \sum_{1 \le j < k \le N} V^{(jk)}(|\vec{r}_j - \vec{r}_k|)$$

is Galilean invariant. Here $v_k = |\dot{\vec{r}}_k|$ and $V^{(jk)}(r)$ is a potential of a pairwise interparticle interaction.

Exercise 2 (JS 11, partial)

Consider are three-dimensional one-particle system whose potential energy in cylindrical polar coordinates ρ, θ, z is of the form $V(\rho, k\theta + z)$, where k is a constant.

a) Find a symmetry of the Lagrangian and use Noether's theorem to obtain the constant of motion associated with it.

b) Write down at least one other constant of the motion (e.g. energy).

*c) Can you give a "natural" example of a system with such a potential?

Exercise 3

Consider a three-dimensional one-particle system whose potential energy in Cartesian coordinates is $V(x, y, z) = A\cos(x-2y) + B\sin(y+3z) + C\cos(x+6z)$, where A, B, C are constants.

a) Find a symmetry of the Lagrangian and use Noether's theorem to obtain the constant of motion associated with it.

b) Write down at least one other constant of the motion (e.g. energy).

Exercise 4

Consider a three-dimensional one-particle system whose Lagrangian is given by

$$L = \frac{1}{2}mv^2 + \frac{1}{2}e\vec{v} \cdot \left[\vec{B} \times \vec{r}\right],$$

where $\vec{B}$ is a constant vector (magnetic field).

a) Find at least on symmetry of the Lagrangian (other then energy) and use Noether's theorem to obtain the constant of motion associated with it.

b) Write down the expression for a conserved energy of the system.

Exercise 5

Consider a particle of the mass m moving on a hyperbolic plane. One can think of the hyperbolic plane as of a unit disk $u^2+v^2 \leq 1$ (disk model of a hyperbolic plane) with the metric

$$dl^2 = \frac{du^2+dv^2}{(1-u^2-v^2)^2}.$$

a) Write down the Lagrangian of this particle.

b) Find the expression for the conserved energy in the problem.

c) Find the Lagrangian and the energy of the problem in "polar" coordinates r, ϕ:

$$\begin{aligned} u &= r\cos\phi, \\ v &= r\sin\phi. \end{aligned}$$

d) Find the integral of motion (angular momentum) corresponding to the rotational symmetry of the Lagrangian.

*e) Using the results of c) and d) write down the first order differential equation of motion for $r(t)$.

*f) Show that this Lagrangian (metric) is invariant with respect to the symmetry

$$z \to e^{i\alpha}\frac{z-a}{1-za^*},$$

where $z = u+iv$, $0 \leq \alpha < 2\pi$, and $|a| < 1$.

*g) Real α and complex a define three parameter symmetry group of the problem. Find three integrals of motion corresponding to those parameters.