9/19/2011. **Due:** Monday, Sep. 26, 2011 (in class).

Physics 501: Classical Mechanics

Read: **LL** 10-15; **G** 3; **JS** 1.5; **FW** 1.

Homework 3

Exercise 1

The ideal (flexible, uniform, frictionless, etc.) rope of the length l and mass M starts sliding off the ideal frictionless table as shown in the figure (the rope is initially at rest, the gravitational acceleration is g).

a) Introduce some generalized coordinate and write down the Lagrangian of the system.

b) Suppose that it takes time T for the rope to slide off the table completely. Use scaling arguments to find this time for the rope of the length $l' = 2l$.

c) Calculate the time T.

d) What is the magnitude and the direction of the reaction force of the table?

Hint: in d) calculate the rate of the change of the total momentum of the rope.

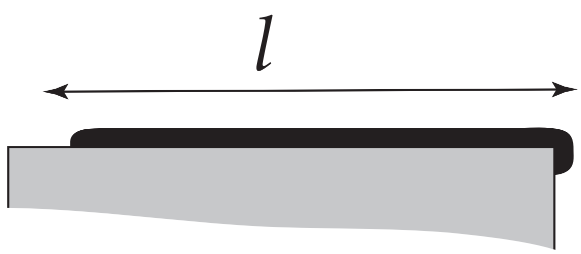

Figure 1: To Exercise 1.

Exercise 2 (JS 1.19, 1.21)

Draw the phase portrait for a particle of mass m moving in

a) a uniform gravitational field. Make this a system of one degree of freedom by considering motion only in the vertical direction.

b) one dimension under the influence of the force $F = -kx + \frac{a}{x^3}$.

Exercise 3

Two blocks of masses m_1 and m_2 are connected by the spring of the equilibrium length l and spring constant k. The initial positions of blocks on a frictionless table are given by $(x, y) = (0, 0)$ and $(0, l)$ respectively. At the moment $t = 0$ the block m_2 receives an instantaneous kick which sets its velocity to be $\vec{v} = v\hat{x}$.

a) Write down the Lagrangian of the system.

b) Reduce this two-body problem to a one-body one.

c) For an effective one-body problem write down the conserved quantities.

d) What is the minimal and the maximal distance between blocks during the motion.

e) Find the positions of the blocks at time t. It is enough to solve the problem in quadratures.

Exercise 4 (FW 1.12)

The orbit of the planet Mercury has an eccentricity 0.206 and a period 0.241 year; moreover, the perihelion advances slowly at the rate of 43 seconds of arc per century. On possible explanation of this effect is that the potential energy around the Sun has the form $V = -(mMG/r)(1 + \alpha GM/rc^2)$, where α is a dimensionless constant and $MG/c^2 \approx 1.475$ km characterizes the Sun's gravitational field. Demonstrate that the resulting orbit indeed represents a precessing ellipse. Find the magnitude and sign of α needed to fit the observed data.

Hint: You may use the solution of the problem 3 after LL 15.